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ABJM models in $\mathcal{N} = 3$ harmonic superspace

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Abstract: We construct the classical action of the Aharony-Bergman-Jafferis-Maldacena (ABJM) model in the $\mathcal{N}$=3, d=3 harmonic superspace. In such a formulation three out of six supersymmetries are realized off shell while the other three mix the superfields and close on shell. The superfield action involves two hypermultiplet superfields in the bifundamental representation of the gauge group and two Chern-Simons gauge superfields corresponding to the left and right gauge groups. The $\mathcal{N}$=3 superconformal invariance allows only for a minimal gauge interaction of the hypermultiplets. Amazingly, the correct sextic scalar potential of ABJM emerges after the elimination of auxiliary fields. Besides the original $\mathrm{U}(N)\times\mathrm{U}(N)$ ABJM model, we also construct $\mathcal{N}$=3 superfield formulations of some generalizations. For the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ case we give a simple superfield proof of its enhanced $\mathcal{N}$=8 supersymmetry and SO(8) R-symmetry.

Keywords: Extended Supersymmetry, Superspaces, Chern-Simons Theories, Supersymmetric gauge theory

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Contents

1 Introduction

The last year has witnessed impressive progress in constructing the actions of multiple M2 branes and studying their properties. M2 branes can be described by three-dimensional superconformal field theories, which have the structure of Chern-Simons-matter theory with $\mathcal{N}$=6 or $\mathcal{N}$=8 extended supersymmetry. The problem of constructing such actions

JHEP03(2009)096

for multiple M2 branes was raised several years ago in [1], but was resolved only recently in a series of works [2–6]. Various aspects of these theories were studied subsequently; a partial list of papers is [7]–[37].

Of special interest is the work of Aharony, Bergman, Jafferis and Maldacena (ABJM) [5] in which the three-dimensional $\mathcal{N}$=6 superconformal theory was constructed and proved to describe multiple M2 branes on the $\mathbb{C}^4/\mathbb{Z}_k$ orbifold. The ABJM model plays a fundamental role, since many three-dimensional superconformal theories such as the Bagger-Lambert-Gustavsson (BLG) model with maximal $\mathcal{N}$=8 supersymmetry [2, 3] and other models with less supersymmetry follow from the ABJM one under particular choices of the gauge group. The field content of the ABJM model is given by four complex scalar and spinor fields which live in the bifundamental representation of the $\mathrm{U}(N)\times\mathrm{U}(N)$ gauge group[1] while the gauge fields are governed by Chern-Simons actions of levels k and $-k$, respectively.

It is desirable to have a superfield description of the ABJM models, with maximal number of manifest and off-shell supersymmetries. As in other cases, such superfield formulations are expected to bring to light geometric and quantum properties of the theory which are implicit in the component formulation. To date, several approaches to the superfield description of the ABJM and BLG theories are known. They use either $\mathcal{N}$=1 and $\mathcal{N}$=2 off-shell superfields [7, 31, 32] or $\mathcal{N}$=6 and $\mathcal{N}$=8 on-shell superfields [33, 35]. These formulations were able to partly clarify the origin of the interaction of scalar and spinor component fields.[2]

In the present paper we take the next step in working out off-shell superfield formulations of the ABJM theory. Namely, we develop its formulation in $\mathcal{N}$=3, d=3 harmonic superspace, which was proposed in [40, 41] as the appropriate adaptation of the $\mathcal{N}$=2, d=4 harmonic superspace [42, 43]. The four complex scalars and spinors are embedded into two q hypermultiplet analytic superfields which sit in the bifundamental representation of the $\mathrm{U}(N)\times\mathrm{U}(N)$ gauge group. The gauge part of the action is given by a sum of two $\mathcal{N}$=3 supersymmetric Chern-Simons actions with levels k and $-k$, respectively, just as in the component approach [5]. In this formulation, three out of six supersymmetries are realized off shell and are manifest, while the other three transform the gauge superfields and hypermultiplets into each other and close only on shell. The same concerns the full automorphism group $\mathrm{SO}(6)\sim\mathrm{SU}(4)$ of the $\mathcal{N}$=6 supersymmetry: only its $\mathrm{SU}(2)\times\mathrm{SU}(2)$ subgroup is manifest in the $\mathcal{N}$=3 superfield formalism, while the coset $\mathrm{SU}(4)/[\mathrm{SU}(2)\times\mathrm{SU}(2)]$ is realized by nonlinear superfield transformations with an on-shell closure.[3]

The scale invariance of the ABJM theory imposes severe restrictions on the action in the $\mathcal{N}$=3 superfield formulation: only minimal interactions of the q hypermultiplets with the gauge superfields are admissible, and no explicit superpotential can be constructed.

[1] Generalizations to some other gauge groups are described, e.g., in [16, 28–30].

[2] Earlier important references on superfield extensions of Chern-Simons theory with and without matter couplings are [38, 39] and [40].

[3] The pure Chern-Simons theory also admits off-shell $\mathcal{N}$=5 and $\mathcal{N}$=6 extensions in some specific harmonic superspaces [44, 45]. However, it is likely that analogous superextensions of the Chern-Simons-matter systems do not exist, rendering the $\mathcal{N}$=3 extension as the *maximal off-shell* one.

JHEP03(2009)096

One may wonder how the sextic scalar potential of the ABJM model can appear in the absence of an original superpotential. We show that, upon reducing the superfield action to the component form, the scalar potential naturally arises as a result of eliminating some auxiliary fields from the gauge multiplet and from the harmonic expansion of the off-shell q hypermultiplets. This is a striking new feature of the $\mathcal{N}$=3 superfield formulation as compared to the $\mathcal{N}$=1 and $\mathcal{N}$=2 ones.

The paper is organized as follows. In section 2 we collect the basic building blocks of the $\mathcal{N}$=3, d=3 harmonic superspace approach which are used in section 3 for constructing the $\mathcal{N}$=3 superfield action of the ABJM model and for demonstrating its $\mathcal{N}$=6 and SO(6) (super)invariances, for the gauge group $\mathrm{U}(N)\times\mathrm{U}(M)$. We also show how the sextic scalar potential of the ABJM model emerges. In section 4 we present $\mathcal{N}$=3 superfield formulations for a variant of the ABJM theory with gauge group $\mathrm{SO}(N)\times\mathrm{USp}(2M)$, which respects $\mathcal{N}$=5 supersymmetry and SO(5) R-symmetry. We also demonstrate in a simple way that the $\mathrm{SU}(N)\times\mathrm{SU}(M)$ model admits hidden supersymmetry and R-symmetry ($\mathcal{N}$=6 and SO(6)) only for the choice $N{=}M$. Section 5 is devoted to the special $\mathrm{SU}(2)\times\mathrm{SU}(2)$ case in which the ABJM model coincides with the BLG one. We present in $\mathcal{N}$=3 superfield form the hidden $\mathcal{N}$=8 supersymmetry and SO(8) R-symmetry of this model. The final section 6 contains a discussion of our results and marks prospects of their applications to M2 branes and their relation with D2 branes. In an appendix, for the simple example of the $\mathrm{U}(1)\times\mathrm{U}(1)$ model, we describe the $\mathcal{N}$=3 superfield realization of the Higgs-type effect of [37] which relates M2 branes to D2 branes.

2 Gauge and matter theories in $\mathcal{N}$=3, d=3 harmonic superspace

2.1 Superspace conventions

We start with a short review of the $\mathcal{N}$=3, d=3 harmonic superspace and field models therein which were originally introduced in [40, 41]. Our three-dimensional notations are as follows: we use the Greek letters $\alpha,\beta,\ldots$ to label the spinorial indices corresponding to the $\mathrm{SO}(1,2)\simeq\mathrm{SL}(2,R)$ Lorentz group. A vector in d=3 Minkowski space is equivalent to a second-rank symmetric spinor, $x^{\alpha\beta}=x^m(\gamma_m)^{\alpha\beta}$

$$\begin{aligned}(\gamma_m)^\rho_\alpha(\gamma_n)^\beta_\rho &= -(\gamma_m)_{\alpha\rho}(\gamma_n)^{\rho\beta} = -\eta_{mn}\delta^\beta_\alpha + \varepsilon_{mnp}(\gamma^p)^\beta_\alpha,\\ (\gamma^m)^{\alpha\beta}(\gamma_m)_{\rho\sigma} &= 2\delta^\alpha_{(\rho}\delta^\beta_{\sigma)}\,,\end{aligned} \tag{2.1}$$

where $\eta_{mn}=\mathrm{diag}(1,-1,-1)$ is the d=3 Minkowski metric. The R-symmetry of $\mathcal{N}$=3 superspace is $\mathrm{SO}(3)_R\simeq\mathrm{SU}(2)_R$. Therefore we label the three copies of Grassmann variables by a pair of symmetric SU(2) indices i,j, i.e., $\theta^{ij}_\alpha=\theta^{ji}_\alpha$. Hence, the $\mathcal{N}$=3 superspace is parametrized by the following real coordinates

$$z=(x^m,\theta^{ij}_\alpha),\quad \overline{x^m}=x^m,\quad \overline{\theta^{ij}_\alpha}=\theta_{ij\alpha}\,. \tag{2.2}$$

The partial spinor and vector derivatives are defined as follows

$$\frac{\partial}{\partial\theta^{ij}_\alpha}\theta^{kl}_\beta=\delta^\alpha_\beta\,\delta^k_{(i}\delta^l_{j)}\,,\quad \partial_{\alpha\beta}x^{\rho\sigma}=2\delta^\rho_{(\alpha}\delta^\sigma_{\beta)}\,,\quad \partial_{\alpha\beta}=(\gamma^m)_{\alpha\beta}\frac{\partial}{\partial x^m}\,. \tag{2.3}$$

JHEP03(2009)096

These derivatives are used to construct covariant spinor derivatives and supercharges,

$$D^{kj}_\alpha = \frac{\partial}{\partial\theta^\alpha_{kj}} + i\theta^{kj\,\beta}\partial_{\alpha\beta}, \quad Q^{kj}_\alpha = \frac{\partial}{\partial\theta^\alpha_{kj}} - i\theta^{kj\,\beta}\partial_{\alpha\beta}\,. \tag{2.4}$$

The spinor indices as well as the R-symmetry ones are raised and lowered with the anti-symmetric two-dimensional tensors $\varepsilon_{\alpha\beta}$, ε_{ij}, respectively ($\varepsilon_{12} = -\varepsilon^{12} = 1$).

We use standard harmonic variables $u^\pm_i$ parametrizing the coset SU(2)/U(1) [42, 43]. In particular, the partial harmonic derivatives are

$$\partial^{++} = u^+_i \frac{\partial}{\partial u^-_i}, \quad \partial^{--} = u^-_i \frac{\partial}{\partial u^+_i}, \quad \partial^0 = [\partial^{++}, \partial^{--}] = u^+_i \frac{\partial}{\partial u^+_i} - u^-_i \frac{\partial}{\partial u^-_i}. \tag{2.5}$$

The harmonic projections of the Grassmann $\mathcal{N}$=3 coordinates and spinor derivatives can be defined as follows

$$\begin{aligned} \theta^{ij}_\alpha &\longrightarrow (\theta^{++}_\alpha, \theta^{--}_\alpha, \theta^0_\alpha) = (u^+_i u^+_j \theta^{ij}_\alpha, u^-_i u^-_j \theta^{ij}_\alpha, u^+_i u^-_j \theta^{ij}_\alpha), \\ D^{ij}_\alpha &\longrightarrow (D^{++}_\alpha, D^{--}_\alpha, D^0_\alpha) = (u^+_i u^+_j D^{ij}_\alpha, u^-_i u^-_j D^{ij}_\alpha, u^+_i u^-_j D^{ij}_\alpha). \end{aligned} \tag{2.6}$$

The analytic subspace in the full $\mathcal{N}$=3 superspace is parametrized by the following coordinates:

$$\zeta_A = (x^{\alpha\beta}_A, \theta^{++}_\alpha, \theta^0_\alpha, u^\pm_i), \tag{2.7}$$

where

$$x^{\alpha\beta}_A = (\gamma_m)^{\alpha\beta} x^m_A = x^{\alpha\beta} + i(\theta^{++\alpha}\theta^{--\beta} + \theta^{++\beta}\theta^{--\alpha}). \tag{2.8}$$

It is instructive to rewrite the harmonic and Grassmann derivatives in the analytic coordinates,

$$\begin{aligned} \mathcal{D}^{++} &= \partial^{++} + 2i\theta^{++\alpha}\theta^{0\beta}\partial^A_{\alpha\beta} + \theta^{++\alpha}\frac{\partial}{\partial\theta^{0\alpha}} + 2\theta^{0\alpha}\frac{\partial}{\partial\theta^{--\alpha}}, \\ \mathcal{D}^{--} &= \partial^{--} - 2i\theta^{--\alpha}\theta^{0\beta}\partial^A_{\alpha\beta} + \theta^{--\alpha}\frac{\partial}{\partial\theta^{0\alpha}} + 2\theta^{0\alpha}\frac{\partial}{\partial\theta^{++\alpha}}, \\ \mathcal{D}^0 &= \partial^0 + 2\theta^{++\alpha}\frac{\partial}{\partial\theta^{++\alpha}} - 2\theta^{--\alpha}\frac{\partial}{\partial\theta^{--\alpha}}, \quad [\mathcal{D}^{++}, \mathcal{D}^{--}] = \mathcal{D}^0, \end{aligned} \tag{2.9}$$

$$D^{++}_\alpha = \frac{\partial}{\partial\theta^{--\alpha}}, \quad D^{--}_\alpha = \frac{\partial}{\partial\theta^{++\alpha}} + 2i\theta^{--\beta}\partial^A_{\alpha\beta}, \quad D^0_\alpha = -\frac{1}{2}\frac{\partial}{\partial\theta^{0\alpha}} + i\theta^{0\beta}\partial^A_{\alpha\beta}, \tag{2.10}$$

where $\partial^A_{\alpha\beta} = (\gamma^m)_{\alpha\beta}\partial/\partial x^m_A$. These derivatives satisfy the following relations:

$$\{D^{++}_\alpha, D^{--}_\beta\} = 2i\partial^A_{\alpha\beta}, \qquad \{D^0_\alpha, D^0_\beta\} = -i\partial^A_{\alpha\beta}, \qquad \{D^{\pm\pm}_\alpha, D^0_\beta\} = 0\,, \tag{2.11}$$

$$[\mathcal{D}^{\mp\mp}, D^{\pm\pm}_\alpha] = 2D^0_\alpha, \qquad [\mathcal{D}^0, D^{\pm\pm}_\alpha] = \pm 2D^{\pm\pm}_\alpha, \qquad [\mathcal{D}^{\pm\pm}, D^0_\alpha] = D^{\pm\pm}_\alpha. \tag{2.12}$$

The analytic superfields are defined to be independent of the θ^{--}_α variable

$$D^{++}_\alpha \Phi_A = 0 \quad \Rightarrow \quad \Phi_A = \Phi_A(\zeta_A)\,. \tag{2.13}$$

We use the following conventions for the full and analytic integration measures,

$$d^9z = -\frac{1}{16}d^3x(D^{++})^2(D^{--})^2(D^0)^2, \tag{2.14}$$

$$d\zeta^{(-4)} = \frac{1}{4}d^3x_A du(D^{--})^2(D^0)^2\,, \quad d^9z du = -\frac{1}{4}d\zeta^{(-4)}(D^{++})^2\,, \tag{2.15}$$

JHEP03(2009)096

where $(D^{++})^2 = D^{++\alpha}D^{++}_\alpha$ and similarly for other objects. With such conventions, the superspace integration rules are most simple:

$$\int d\zeta^{(-4)}(\theta^{++})^2(\theta^0)^2 f(x_A) = \int d^3x_A f(x_A)\,,$$
$$\int d^9z(\theta^{++})^2(\theta^{--})^2(\theta^0)^2 f(x) = \int d^3x\, f(x)\,, \tag{2.16}$$

for some field $f(x)$.

We denote the special conjugation in the $\mathcal{N}$=3 harmonic superspace by $\widetilde{\ }$

$$\widetilde{(u^\pm_i)} = u^{\pm i}, \quad \widetilde{(x^m_A)} = x^m_A, \quad \widetilde{(\theta^{\pm\pm}_\alpha)} = \theta^{\pm\pm}_\alpha, \quad \widetilde{(\theta^0_\alpha)} = \theta^0_\alpha. \tag{2.17}$$

It is squared to -1 on the harmonics and to 1 on x^m_A and Grassmann coordinates. All bilinear combinations of the Grassmann coordinates are imaginary

$$[\widetilde{(\theta^{++}_\alpha\theta^0_\beta)}] = -\theta^{++}_\alpha\theta^0_\beta, \quad [\widetilde{(\theta^{++})^2}] = -(\theta^{++})^2, \quad [\widetilde{(\theta^0)^2}] = -(\theta^0)^2. \tag{2.18}$$

The conjugation rules for the spinor and harmonic derivatives are

$$\widetilde{(D^0_\alpha\Phi)} = -D^0_\alpha\tilde\Phi, \quad [\widetilde{(D^0)^2\Phi}] = -(D^0)^2\tilde\Phi, \quad \widetilde{(\mathcal{D}^{++}\Phi)} = \mathcal{D}^{++}\tilde\Phi \tag{2.19}$$

where Φ and $\tilde\Phi$ are conjugated even superfields. When the superfields are matrix-like objects, $\Phi = [\Phi^A_B]$, the Hermitian conjugation assumes the $\widetilde{\ }$ conjugation and transposition, e.g., $[\Phi^A_B]^\dagger = \widetilde\Phi^B_A$.

The analytic superspace measure is real, $\widetilde{d\zeta^{(-4)}} = d\zeta^{(-4)}$, while the full superspace measure is imaginary, $\widetilde{d^9z} = -d^9z$.

2.2 Chern-Simons and hypermultiplet actions in $\mathcal{N}$=3 harmonic superspace

2.2.1 Chern-Simons action

The $\mathcal{N}$=3 supersymmetric gauge multiplet in three dimensions consists of a triplet of real scalar fields $\phi^{(kl)}$, one real vector A_m, real SU(2)-singlet spinor λ_α, SU(2)-triplet spinor $\chi^{(kl)}_\alpha$ and a triplet of auxiliary fields $X^{(kl)}$. Altogether they constitute eight bosonic and eight fermionic off-shell degrees of freedom. All these components are embedded into an analytic gauge superfield which originally contains an infinite set of fields in its θ and u-expansion. However, like in the $\mathcal{N}$=2 , d=4 case [42, 43], the gauge freedom with an analytic superfield parameter allows one to pass to the Wess-Zumino gauge which reveals the above finite irreducible field content of the $\mathcal{N}$=3 gauge multiplet

$$\begin{aligned} V^{++}_{\rm WZ} = {}& 3(\theta^{++})^2u^-_ku^-_l\phi^{kl}(x_A) + 2\theta^{++\alpha}\theta^{0\beta}A_{\alpha\beta}(x_A) + 2(\theta^0)^2\theta^{++\alpha}\lambda_\alpha(x_A) \\ & + 3(\theta^{++})^2\theta^{0\alpha}u^-_ku^-_l\chi^{kl}_\alpha(x_A) + 3i(\theta^{++})^2(\theta^0)^2u^-_ku^-_lX^{kl}(x_A). \end{aligned} \tag{2.20}$$

In the Abelian case the corresponding gauge transformation of the imaginary superfield V^{++} reads

$$\delta_\Lambda V^{++} = -\mathcal{D}^{++}\Lambda, \qquad \widetilde\Lambda = -\Lambda, \tag{2.21}$$

The non-Abelian gauge superfield has the following infinitesimal transformation law

$$\delta_\Lambda V^{++} = -\mathcal{D}^{++}\Lambda - [V^{++},\Lambda]. \tag{2.22}$$

In what follows we shall be mainly interested in the gauge group U(N) in the fundamental representation and its adjoint. In this case, V^{++} and Λ are antihermitian $N\times N$ matrices

$$[\widetilde{V^{++}{}^B_A}] = -V^{++}{}^A_B, \quad [\widetilde{\Lambda^B_A}] = -\Lambda^A_B \quad A,B=1,2,\ldots,N\,. \tag{2.23}$$

The SU(N) case is singled out by the extra tracelessness condition

$$V^{++}{}^A_A = \Lambda^A_A = 0\,. \tag{2.24}$$

Using V^{++}, one can construct either the Yang-Mills or Chern-Simons actions in the $\mathcal{N}$=3 superspace [40, 41]. The non-Abelian Chern-Simons superfield action is

$$S_{\rm CS} = \frac{ik}{4\pi}\,{\rm tr}\sum_{n=2}^{\infty}\frac{(-1)^n}{n}\int d^3x d^6\theta du_1\ldots du_n\frac{V^{++}(z,u_1)\ldots V^{++}(z,u_n)}{(u_1^+u_2^+)\ldots(u_n^+u_1^+)}, \tag{2.25}$$

where k is the Chern-Simons level. Note that this action is formally analogous to the superfield action of the $\mathcal{N}$=2, d=4 Yang-Mills theory [46], although the full integration measure is $d^4xd^8\theta$ in the latter case. The action (2.25) can be checked to be invariant under the gauge transformation (2.22).

For what follows it will be necessary to know a general variation of the Chern-Simons action (2.25)

$$\delta S_{\rm CS} = -\frac{ik}{4\pi}{\rm tr}\int d^9z du\,\delta V^{++}V^{--}\,. \tag{2.26}$$

Here V^{--} is the non-analytic gauge superfield which is related to V^{++} by the harmonic zero-curvature equation [43, 46]

$$\mathcal{D}^{++}V^{--} - \mathcal{D}^{--}V^{++} + [V^{++},V^{--}] = 0 \tag{2.27}$$

and is transformed under the gauge group as

$$\delta_\Lambda V^{--} = -\mathcal{D}^{--}\Lambda - [V^{--},\Lambda]\,. \tag{2.28}$$

The solution of (2.27) is represented by the following series

$$V^{--}(z,u) = \sum_{n=1}^{\infty}(-1)^n\int du_1\ldots du_n\frac{V^{++}(z,u_1)V^{++}(z,u_2)\ldots V^{++}(z,u_n)}{(u^+u_1^+)(u_1^+u_2^+)\ldots(u_n^+u^+)}. \tag{2.29}$$

The superfield V^{--} can be used to define the superfield strength W^{++} [41],

$$W^{++} = -\frac{1}{4}D^{++\alpha}D^{++}_\alpha V^{--}, \qquad \mathcal{D}^{++}W^{++} + [V^{++},W^{++}] = 0. \tag{2.30}$$

By construction, W^{++} is analytic and gauge covariant, $\delta_\Lambda W^{++} = [\Lambda, W^{++}]$. Note that W^{++} is hermitian in contrast to the gauge superfield V^{++}, $(W^{++})^\dagger = W^{++}$. In terms of W^{++}, the variation (2.26) of the Chern-Simons action can be written as

$$\delta S_{\rm CS} = -\frac{ik}{4\pi}{\rm tr}\int d\zeta^{(-4)}\delta V^{++}W^{++}\,. \tag{2.31}$$

The classical equation of motion in the pure super Chern-Simons model is $W^{++}=0$, which implies the superfields $V^{\pm\pm}$ to be pure gauge. The topological character of the $\mathcal{N}$=3 gauge multiplet with the Chern-Simons action (2.25) can also be seen directly from the component structure of this action:[4]

$$\begin{aligned} S_{\rm CS} = \frac{k}{4\pi}{\rm tr}\int d^3x\Big(\phi^{kl}X_{kl} - \frac{2i}{3}\phi^i_j[\phi^k_i,\phi^j_k] + \frac{i}{2}\lambda^\alpha\lambda_\alpha - \frac{i}{4}\chi^\alpha_{kl}\chi^{kl}_\alpha \\ -\frac{1}{2}A^{\alpha\beta}\partial^\gamma_\alpha A_{\beta\gamma} - \frac{i}{6}A^\alpha_\beta[A^\gamma_\alpha, A^\beta_\gamma]\Big). \end{aligned} \tag{2.32}$$

Using d=3 γ-matrices one can convert the vector part of the action (2.32) to the standard form $\varepsilon^{mnp}(A_m\partial_n A_p - \frac{2i}{3}A_mA_nA_p)$.

As for the $\mathcal{N}$=3, d=3 super Yang-Mills action, it is concisely written as the following integral over the analytic superspace

$$S_{\rm SYM} = -\frac{1}{g^2}\,{\rm tr}\int d\zeta^{(-4)}\,(W^{++})^2\,,\quad [g]=1/2\,. \tag{2.33}$$

It should be compared with the $\mathcal{N}$=2, d=4 SYM action in the harmonic superspace which is represented either by the action of the type (2.25) or as an integral of the square of the relevant chiral (ant-chiral) superfield strength over the chiral (anti-chiral) $\mathcal{N}$=2, d=4 superspace [42, 43].

2.2.2 Hypermultiplet action

Like in the $\mathcal{N}$=2, d=4 case [42, 43], the $\mathcal{N}$=3, d=3 hypermultiplet is described by an analytic harmonic superfield $q^+(\zeta)$ with the following free action

$$S_q = \int d\zeta^{(-4)}\bar q^+\mathcal{D}^{++}q^+\,,\quad \bar q^+ = \widetilde{q^+}\,,\quad \widetilde{\bar q^+} = -q^+\,. \tag{2.34}$$

The physical fields of $\mathcal{N}$=3, d=3 hypermultiplet are SU(2) doublets f^i and ψ^i_α. After elimination of an infinite tower of auxiliary fields by their equations of motion (they all vanish on shell) the physical fields appear in the component expansion of the analytic superfield q^+ and its conjugated $\bar q^+$ as

$$\begin{aligned} q^+ &= u^+_i f^i + (\theta^{++\alpha}u^-_i - \theta^{0\alpha}u^+_i)\psi^i_\alpha - 2i(\theta^{++\alpha}\theta^{0\beta})\partial^A_{\alpha\beta}f^iu^-_i\,, \\ \bar q^+ &= -u^+_i\bar f^i + (\theta^{++\alpha}u^-_i - \theta^{0\alpha}u^+_i)\bar\psi^i_\alpha + 2i(\theta^{++\alpha}\theta^{0\beta})\partial^A_{\alpha\beta}\bar f^iu^-_i\,. \end{aligned} \tag{2.35}$$

All component fields are defined on the d=3 Minkowski space x^m_A. With the auxiliary fields being eliminated, the superfield action (2.34) yields the following action for the physical fields:

$$S_{\rm phys} = -\int d^3x\Big(\bar f_i\Box f^i + \frac{i}{2}\bar\psi^\alpha_i\partial_{\alpha\beta}\psi^{i\beta}\Big). \tag{2.36}$$

Note that the presence of an infinite number of the auxiliary fields is an unavoidable feature of the formulation of the d=3 hypermultiplets with off-shell $\mathcal{N}$=3 supersymmetry, in a full similarity to off-shell $\mathcal{N}$=2, d=4 hypermultiplets.

[4] The component structure of the $\mathcal{N}$=3 Chern-Simons action with the matter couplings added was given in [47].

When the superfield q^+ is placed in some representation of the gauge group,

$$\delta q^+ = \Lambda q^+ \,, \tag{2.37}$$

its minimal coupling to the gauge superfield V^{++} is given by

$$S = \int d\zeta^{(-4)} \bar{q}^+ (\mathcal{D}^{++} + V^{++}) q^+ . \tag{2.38}$$

At the moment we do not specify neither gauge group nor representation of the latter on q^+; the specific cases we shall consider in the next sections correspond to some detailing of the general gauged action (2.38).

2.3 $\mathcal{N}$=3 superconformal transformations

It is easy to construct the odd part of the $\mathcal{N}$=3 superconformal transformations of the coordinates of the initial $\mathcal{N}$=3 superspace:

$$\begin{aligned}
\delta_{\rm sc} x^{\alpha\beta} &= -i\epsilon^{kl\beta}\theta^\alpha_{kl} - i\epsilon^{kl\alpha}\theta^\beta_{kl} \\
&\quad -\frac{i}{2}\eta^{kl}_\gamma \theta^\alpha_{kl} x^{\gamma\beta} - \frac{i}{2}\eta^{kl}_\gamma \theta^\beta_{kl} x^{\gamma\alpha} + \frac{1}{2}\eta^{kl}_\rho \theta^\alpha_{kl}\theta^{jn\rho}\theta^\beta_{jn} + \frac{1}{2}\eta^{kl}_\rho \theta^\beta_{kl}\theta^{jn\rho}\theta^\alpha_{jn}, \\
\delta_{\rm sc}\theta^\alpha_{kl} &= \epsilon^\alpha_{kl} + \frac{1}{2}x^{\alpha\beta}\eta_{kl\beta} - i\theta^\alpha_{jn}\theta^\gamma_{kl}\eta^{jn}_\gamma + \frac{i}{2}\theta^{jn\alpha}\theta^\beta_{jn}\eta_{kl\beta},
\end{aligned} \tag{2.39}$$

where ϵ^α_{kl} and η^α_{kl} are parameters of Q and S supersymmetries. All even superconformal transformations are contained in the Lie brackets of these odd transformations. The full measure $d^3x d^6\theta$ is invariant under the $\mathcal{N}$=3 superconformal group.

The superconformal transformations of the harmonics can be defined by analogy with the $\mathcal{N}$=2, d=4 case [43],

$$\delta_{\rm sc} u^+_k = \lambda^{++} u^-_k \,, \qquad \delta_{\rm sc} u^-_k = 0 \,, \tag{2.40}$$

where

$$\lambda^{++} = -i\theta^{++\alpha}\theta^{0\beta}k_{\alpha\beta} - i\theta^{++\alpha}u^+_k u^-_l \eta^{kl}_\alpha + i\theta^{0\alpha}u^+_k u^+_l \eta^{kl}_\alpha + u^+_k u^+_l \omega^{kl} \,. \tag{2.41}$$

Here $k_{\alpha\beta}$ and ω^{kl} are parameters of the special conformal and SU$(2)_c$ transformations. The transformations of the analytic $\mathcal{N}$=3 coordinates under the S supersymmetry and SU$(2)_c$ symmetry are

$$\begin{aligned}
\delta_{\rm sc} x^m_A &= -i(\gamma^m)_{\alpha\beta}[\, x^{\beta\rho}_A u^-_k u^-_l \eta^{kl}_\rho \theta^{++\alpha} - x^{\rho\beta}_A u^+_k u^-_l \eta^{kl}_\rho \theta^{0\alpha} - 2\omega^{kl} u^-_k u^-_l \theta^{(\alpha\, ++}\theta^{\beta)0}\,] \,, \\
\delta_{\rm sc}\theta^{0\alpha} &= \frac{1}{2}x^{\alpha\beta}_A u^+_k u^-_l \eta^{kl}_\beta - i u^-_k u^-_l \eta^{kl}_\gamma \theta^{++\alpha}\theta^{0\gamma} - \frac{i}{2}u^+_k u^-_l \eta^{kl\alpha}(\theta^0)^2 + \omega^{kl}u^-_k u^-_l \theta^{++\alpha} \,, \\
\delta_{\rm sc}\theta^{++\alpha} &= \frac{1}{2}x^{\alpha\beta}_A u^+_k u^+_l \eta^{kl}_\beta + \frac{i}{2}\eta^{kl\alpha}[u^-_k u^-_l (\theta^{++})^2 - u^+_k u^+_l (\theta^0)^2] + 2\omega^{kl}u^-_k u^+_l \theta^{++\alpha} .
\end{aligned} \tag{2.42}$$

The transformations of the harmonic derivatives have the form

$$\delta_{\rm sc}\mathcal{D}^{++} = -\lambda^{++}\mathcal{D}^0, \quad \delta_{\rm sc}\mathcal{D}^{--} = -(\mathcal{D}^{--}\lambda^{++})\mathcal{D}^{--}. \tag{2.43}$$

It is easy to find the superconformal transformation of the analytic integration measure

$$\delta_{\rm sc} d\zeta^{(-4)} = -2\lambda d\zeta^{(-4)}, \quad \mathcal{D}^{++}\lambda = \lambda^{++} \,. \tag{2.44}$$

Here

$$\lambda = -\frac{1}{2}d - \frac{1}{4}x_A^{\alpha\beta}k_{\alpha\beta} + i\theta^{0\alpha}\eta^0_\alpha - i\theta^{++\alpha}\eta^{--}_\alpha + u^+_k u^-_l \omega^{kl}, \tag{2.45}$$

d being the scale transformation parameter.

The $\mathcal{N}$=3 Chern-Simons action (2.25) and the minimal q^+, V^{++} interaction (2.38) are invariant under the $\mathcal{N}$=3 superconformal group realized on the basic superfields as

$$\delta_{\rm sc}V^{++} = 0, \quad \delta_{\rm sc}q^+ = \lambda q^+. \tag{2.46}$$

The $\mathcal{N}$=3, d=3 action (2.33) is obviously not superconformal because of the presence of dimensionful coupling constant.

For the future use, it is worthwhile to point out that the requirement of superconformal invariance forbids any self-interaction of the hypermultiplets off shell: their only superconformal off-shell actions are the free q^+ action (2.34) and its minimal gauge covariantization (2.38).[5]

3 The ABJM model in $\mathcal{N}$=3 harmonic superspace

3.1 Free hypermultiplets

It is well known that the component content of the $\mathcal{N}$=6 supersymmetric model is given by four complex scalar fields and four complex spinor fields. In the $\mathcal{N}$=3 superfield formalism, these degrees of freedom can be described by two hypermultiplet superfields $q^{+a} = \varepsilon^{ab}q^+_b$, $a, b = 1, 2$, and their conjugate $\bar{q}^+_a = \widetilde{(q^{+a})}$, $\widetilde{(q^+_b)} = -\bar{q}^{+b}$, with the action

$$S_{\rm free} = \int d\zeta^{(-4)}\bar{q}^+_a \mathcal{D}^{++}q^{+a}. \tag{3.1}$$

This action is manifestly invariant under the extra $SU(2)_{\rm ext}$ group acting on the doublet indices a and commuting with the $\mathcal{N}$=3 supersymmetry. It also exhibits an extra U(1) symmetry realized as a common phase transformation of q^{+a}:

$$q^{+a\prime} = e^{i\tau}q^{+a}, \quad \bar{q}^{+\prime}_a = e^{-i\tau}\bar{q}^+_a. \tag{3.2}$$

3.1.1 Extra supersymmetry

The additional (hidden) supersymmetry transformations of the $\mathcal{N}$=3 superfields are defined through the spinor derivative D^0_α preserving the Grassmann analyticity:

$$\delta_\epsilon q^{+a} = i\epsilon^{\alpha\, ab}D^0_\alpha q^+_b = -\widetilde{(\delta_\epsilon \bar{q}^+_a)}, \quad \delta_\epsilon \bar{q}^+_a = i\epsilon^\alpha_{ab}D^0_\alpha \bar{q}^{+b} = \widetilde{(\delta_\epsilon q^{+a})}, \tag{3.3}$$

where $\epsilon^{ab}_\alpha = \epsilon^{ba}_\alpha$ is a real spinor parameter, triplet of the extra SU(2) group, $\widetilde{(\epsilon^{ab}_\alpha)} = \epsilon_{\alpha\, ab}$. Note the conjugation rule

$$\widetilde{(D^0_\alpha q^+_b)} = D^0_\alpha \bar{q}^{+b}.$$

[5]This uniqueness of superconformal q^+ action can be understood also on the dimensionality grounds: the analytic superspace integration measure has dimension -1 (in mass units) while $[q^+] = 1/2$; so the action without dimensionful parameters can be at most bilinear in q^+ superfields.

The free hypermultiplet action (3.1) is easily checked to be invariant under these transformations

$$\delta_\epsilon S_{\rm free} = i\int d\zeta^{(-4)}\epsilon^{\alpha\,ab} D^0_\alpha(\bar q^+_a \mathcal{D}^{++} q^+_b) = 0. \tag{3.4}$$

To show that (3.3) indeed generate supersymmetry, we compute the commutator of two transformations (3.3) with the spinor parameters ϵ^{ab}_α and μ^{ab}_α,

$$\begin{aligned}[]
[\delta_\mu\delta_\epsilon - \delta_\epsilon\delta_\mu] q^{+a} &= -\frac{1}{2}(\mu^{\alpha a}{}_b\epsilon^{\beta\,bc} + \mu^{\beta c}{}_b\epsilon^{\alpha\,ab})\left[\{D^0_\alpha, D^0_\beta\} - \varepsilon_{\alpha\beta}(D^0)^2\right] q^+_c \\
&= \frac{i}{2}\mu^{(\alpha}_{bc}\epsilon^{\beta)\,bc}\partial^A_{\alpha\beta} q^{+a} - \mu^{\alpha(a}{}_b\epsilon^{c)b}_\alpha(D^0)^2 q^+_c.
\end{aligned} \tag{3.5}$$

The last term in (3.5) vanishes on shell, $D^{++}q^+_a = 0 \;\Rightarrow\; (D^0)^2 q^+_a = 0$. As a result, the commutator of two transformations (3.3) generates the x-translations of hypermultiplets with the bosonic parameter $\mu^{(\alpha}_{bc}\epsilon^{\beta)\,bc}$ and, hence, (3.3) do form three supersymmetries on shell. These three additional supersymmetry transformations, together with three explicit $\mathcal{N}$=3 ones, constitute the $\mathcal{N}$=6 invariance of the free hypermultiplet action (3.1). Note that the Lie bracket of the implicit and explicit supersymmetry transformations is vanishing as a consequence of the anticommutativity of D^0_α and the $\mathcal{N}$=3 supersymmetry generators.

3.1.2 SO(6) R-symmetry

The free action of two hypermultiplet superfields also exhibits an invariance under the full automorphism group SO(6) of the $\mathcal{N}$=6 superalgebra.

The action (3.1) is explicitly invariant only under the group $\mathrm{SU}(2)_R\times\mathrm{SU}(2)_{\rm ext}$, where $\mathrm{SU}(2)_R$ is the group of internal automorphisms of $\mathcal{N}$=3 harmonic superspace while $\mathrm{SU}(2)_{\rm ext}$ is realized on the index a in this action. Therefore, to show the invariance of the action under the full SO(6) R-symmetry group we need to specify the remaining transformations from the coset $\mathrm{SO}(6)/[\mathrm{SU}(2)_R\times\mathrm{SU}(2)_{\rm ext}]$. This coset is parametrized by nine real parameters,

$$\lambda^{(ij)(ab)},\quad \overline{(\lambda^{(ij)(ab)})} = \lambda_{(ij)(ab)}. \tag{3.6}$$

The linear realization of these transformations on the physical scalar fields $f^{i\,a}$ can be chosen as

$$\delta_\lambda f^{i\,a} = i\lambda^{(ij)(ab)}f_{j\,b},\quad \delta_\lambda \bar f_{i\,a} = -i\lambda_{(ij)(ab)}\bar f^{j\,b},\quad \bar f_{i\,a} = \overline{f^{i\,a}}, \tag{3.7}$$

so that $f^{i\,a}\bar f_{i\,a}$ is the full SO(6) invariant. These physical scalars appear in the lowest order of the component expansion of the hypermultiplets, $q^{+a} = u^+_i f^{i\,a} + \cdots$, $\bar q^+_a = -u^+_i \bar f^i_a + \cdots$. Therefore there should be a generalization of the transformations (3.7) for the hypermultiplet superfields.

This generalization is unambiguously determined by requiring the variation $\delta_\lambda q^+$ to have the same harmonic U(1) charge +1 as q^+ itself and to be analytic. We project the parameters $\lambda^{(ij)(ab)}$ on the harmonic variables,

$$\lambda^{\pm\pm(ab)} = u^\pm_i u^\pm_j \lambda^{(ij)(ab)},\qquad \lambda^{0(ab)} = u^+_i u^-_j \lambda^{(ij)(ab)}, \tag{3.8}$$

and define the hidden SO(6) transformation of the hypermultiplet superfields as

$$\begin{aligned}
\delta_\lambda q^{+a} &= -i[\lambda^{0(ab)} - \lambda^{++(ab)}\hat{\mathcal{D}}^{--} - 2\lambda^{--(ab)}\theta^{++\alpha}D^0_\alpha + 4\lambda^{0(ab)}\theta^{0\alpha}D^0_\alpha]q^+_b, \\
\delta_\lambda \bar q^+_a &= -i[\lambda^0_{(ab)} - \lambda^{++}_{(ab)}\hat{\mathcal{D}}^{--} - 2\lambda^{--}_{(ab)}\theta^{++\alpha}D^0_\alpha + 4\lambda^0_{(ab)}\theta^{0\alpha}D^0_\alpha]\bar q^{+b}.
\end{aligned} \tag{3.9}$$

Here $\hat{\mathcal{D}}^{--}$ is a modification of the harmonic derivative $\mathcal{D}^{--}$ such that $\hat{\mathcal{D}}^{--}$ preserves analyticity,

$$\hat{\mathcal{D}}^{--} = \mathcal{D}^{--} + 2\theta^{--\alpha} D^0_\alpha = \partial^{--} + 2\theta^{0\alpha}\frac{\partial}{\partial\theta^{++\alpha}}\,, \quad [D^{++}_\alpha, \hat{\mathcal{D}}^{--}] = 0. \tag{3.10}$$

One can easily check that under the superfield transformations (3.9) the lowest bosonic components of the hypermultiplet superfields transform as is (3.7) while the transformations of the auxiliary fields coming from the harmonic expansions are not essential here since these fields vanish on shell.

With the help of the following identity

$$\mathcal{D}^{++}\delta_\lambda q^{+a} = -i[\lambda^{0(ab)} - \lambda^{++(ab)}\hat{\mathcal{D}}^{--} - 2\lambda^{--(ab)}\theta^{++\alpha}D^0_\alpha + 4\lambda^{0(ab)}\theta^{0\alpha}D^0_\alpha]\mathcal{D}^{++}q^+_b\,, \tag{3.11}$$

we compute the variation of the action (3.1),

$$\begin{aligned}\delta_\lambda S_{\text{free}} = -i\int d\zeta^{(-4)}[&2\lambda^0_{(ab)}\bar{q}^{+a}\mathcal{D}^{++}q^{+b} - \lambda^{++}_{(ab)}\hat{\mathcal{D}}^{--}(\bar{q}^{+a}\mathcal{D}^{++}q^{+b})\\ &+4\lambda^0_{(ab)}\theta^{0\alpha}D^0_\alpha(\bar{q}^{+a}\mathcal{D}^{++}q^{+b})].\end{aligned} \tag{3.12}$$

The last term in (3.12) is a total derivative, while after integration by parts the second term cancels the first one. Thus the free hypermultiplet action (3.1) is invariant under (3.9),

$$\delta_\lambda S_{\text{free}} = 0. \tag{3.13}$$

Due to the presence of explicit θs in the transformation (3.9), it does not commute with the manifest $\mathcal{N}$=3 supersymmetry. It is easy to show that, modulo equations of motion for q^{+a}, this commutator yields just the hidden $\mathcal{N}$=3 supersymmetry (3.3). We shall discuss this closure in more detail later on, in the non-trivial interaction cases. It is worth noting that the closure of the hidden SO(6) transformations (3.9) (and their generalization to the interaction case) contains $\mathrm{SU}(2)_{\text{ext}}$ and just the superconformal R-symmetry group $\mathrm{SU}(2)_c$ defined in (2.42). The latter becomes indistinguishable from the standard $\mathrm{SU}(2)_R$ only after elimination of the hypermultiplet auxiliary fields by their equations of motion, i.e. on shell. Note also that the U(1) symmetry (3.2) commutes with both hidden and manifest $\mathcal{N}$=3 supersymmetries (as well as with the extra SO(6) transformations).

In fact, the symmetry of the action (3.1) is even wider than $\mathcal{N}$=6 supersymmetry plus SO(6) R-symmetry: it is the maximal $\mathcal{N}$=8 on-shell supersymmetry together with its automorphism symmetry SO(8). We postpone discussion of these additional symmetries until section 5, where they will be considered at the full interaction level. The off-shell superconformal $\mathcal{N}$=3 invariance of (3.1) taken together with its on-shell SO(8) R-symmetry and $\mathcal{N}$=8 supersymmetry imply that the free action of two d=3 hypermultiplets on shell (i.e. modulo algebraic equations of motion for the auxiliary fields) respects the maximal $\mathcal{N}$=8, d=3 superconformal symmetry.

3.2 The $\mathrm{U}(1)\times\mathrm{U}(1)$ theory

As the next step, we consider the $\mathrm{U}(1)\times\mathrm{U}(1)$ gauge theory. This simplest example with interaction will be used to further explain the basic ideas of our construction.

3.2.1 Actions

Now we have two Abelian gauge superfields V_L^{++} and V_R^{++} corresponding to the two U(1) gauge groups. In accord with the proposal of [5], the gauge action for these superfields should be a difference of two Chern-Simons actions (2.25). In the Abelian case, such action is very simple:

$$\begin{aligned} S_{\rm gauge} &= S_{\rm CS}[V_L^{++}] - S_{\rm CS}[V_R^{++}] \\ &= -\frac{ik}{8\pi}\int d^9z du_1 du_2 \frac{1}{(u_1^+ u_2^+)^2}\left[V_L^{++}(z,u_1)V_L^{++}(z,u_2) - V_R^{++}(z,u_1)V_R^{++}(z,u_2)\right] \\ &= -\frac{ik}{8\pi}\int d\zeta^{(-4)}\left(V_L^{++}W_L^{++} - V_R^{++}W_R^{++}\right), \end{aligned} \tag{3.14}$$

where we used the relation (2.15) and the definition (2.30). The gauge invariant generalization of the hypermultiplet action (3.1) is

$$S_{\rm hyp} = \int d\zeta^{(-4)}\bar q^+_a(\mathcal{D}^{++} + V_L^{++} - V_R^{++})q^{+a} = \int d\zeta^{(-4)}\bar q^+_a \nabla^{++} q^{+a}. \tag{3.15}$$

Note that the gauge covariant harmonic derivative $\nabla^{++} = \mathcal{D}^{++} + V_L^{++} - V_R^{++}$ depends only on the difference of two gauge superfields, but not on their sum (cf. the corresponding covariant space-time derivatives given in [5, 6]). So it is useful to define new gauge superfields,

$$V_L^{++} + V_R^{++} = V^{++}\,, \quad V_L^{++} - V_R^{++} = A^{++}\,, \tag{3.16}$$

in terms of which (3.14) and (3.15) are rewritten as

$$S_{\rm gauge} = -\frac{ik}{8\pi}\int d\zeta^{(-4)}\, V^{++}W^{++}_{(A)} = -\frac{ik}{8\pi}\int d\zeta^{(-4)}\, A^{++}W^{++}_{(V)}\,, \tag{3.17}$$

$$S_{\rm hyp} = \int d\zeta^{(-4)}\bar q^+_a(\mathcal{D}^{++} + A^{++})q^{+a}\,. \tag{3.18}$$

The action (3.18) is invariant under the following gauge transformations

$$q^{+a\prime} = e^{\Lambda}q^{+a}\,, \; \bar q^{+\prime}_a = e^{-\Lambda}\bar q^+_a\,, \; A^{++\prime} = A^{++} - \mathcal{D}^{++}\Lambda\,, \qquad \Lambda = \Lambda_L - \Lambda_R\,. \tag{3.19}$$

The rest of the gauge group $U(1)\times U(1)$, with the gauge parameter $\hat\Lambda = \Lambda_L + \Lambda_R$, acts only on V^{++} and does not affect hypermultiplets at all.

In the considered case, the general variation formula (2.31) is written as

$$\begin{aligned} \delta S_{\rm gauge} &= -\frac{ik}{4\pi}\int d\zeta^{(-4)}\left(\delta V_L^{++}W_L^{++} - \delta V_R^{++}W_R^{++}\right) \\ &= -\frac{ik}{8\pi}\int d\zeta^{(-4)}\left(\delta V^{++}W^{++}_{(A)} + \delta A^{++}W^{++}_{(V)}\right). \end{aligned} \tag{3.20}$$

It is also instructive to present the full set of superfield equations for the $U(1)\times U(1)$ case:

$$\text{(a) } \nabla^{++}q^{+a} = \nabla^{++}\bar q^{+a} = 0; \quad \text{(b) } W_L^{++} = W_R^{++} = -i\frac{4\pi}{k}\bar q^+_a q^{+a}\,. \tag{3.21}$$

The U(1) × U(1) Chern-Simons and hypermultiplet actions are invariant under the P-parity transformation

$$
\begin{aligned}
&Px_A^{0,2} = x_A^{0,2}, \quad Px_A^1 = -x_A^1, \qquad P\theta_\alpha^{0,\pm\pm} = -(\gamma_1)_\alpha^\beta \theta_\beta^{0,\pm\pm}, \quad P(\theta^{0,\pm\pm})^2 = -(\theta^{0,\pm\pm})^2, \\
&PD_\alpha^{0,++} = (\gamma_1)_\alpha^\beta D_\beta^{0,++}, \qquad\qquad P(D^{0,++})^2 = -(D^{0,++})^2.
\end{aligned} \tag{3.22}
$$

The parity of the superfields can be chosen as follows

$$
\begin{aligned}
PV_L^{\pm\pm}(\zeta_P) &= V_R^{\pm\pm}(\zeta), & PW_L^{++}(\zeta_P) &= -W_R^{++}(\zeta), \\
Pq^{+a}(\zeta_P) &= \bar{q}^{+a}(\zeta), & P\bar{q}_a^+(\zeta_P) &= q_a^+(\zeta).
\end{aligned} \tag{3.23}
$$

3.2.2 $\mathcal{N}$=6 supersymmetry

Now we are going to prove that the sum of the gauge and matter actions (3.14), (3.15),

$$
S_{\mathcal{N}=6} = S_{\text{gauge}} + S_{\text{hyp}}\,, \tag{3.24}
$$

possesses the $\mathcal{N}$=6 supersymmetry. To this end, as the first step, we generalize the transformations (3.3),

$$
\begin{aligned}
\delta_\epsilon q^{+a} &= i\epsilon^{\alpha\,(ab)}[\nabla_\alpha^0 + \theta_\alpha^{--}(W_L^{++} - W_R^{++})]q_b^+, \\
\delta_\epsilon \bar{q}_a^+ &= i\epsilon_{(ab)}^\alpha[\nabla_\alpha^0 - \theta_\alpha^{--}(W_L^{++} - W_R^{++})]\bar{q}^{+b}.
\end{aligned} \tag{3.25}
$$

Here ∇_α^0 is a gauge-covariant generalization of the spinor derivative D_α^0. It acts on the hypermultiplets according to

$$
\nabla_\alpha^0 q_a^+ = (D_\alpha^0 + V_{L\,\alpha}^0 - V_{R\,\alpha}^0)q_a^+, \qquad \nabla_\alpha^0 \bar{q}^{+a} = (D_\alpha^0 - V_{L\,\alpha}^0 + V_{R\,\alpha}^0)\bar{q}^{+a}. \tag{3.26}
$$

The gauge potentials $V_{L,R\,\alpha}^0$ are expressed through $V_{L,R}^{--}$ as $V_{L,R\,\alpha}^0 = -\frac{1}{2}D_\alpha^{++}V_{L,R}^{--}$, where $V_L^{--} = V_L^{--}(V_L^{++})$ and $V_R^{--} = V_R^{--}(V_R^{++})$ appear as the solutions of the Abelian version of zero-curvature equation (2.27). In contrast to the flat spinor derivative D_α^0, the gauge-covariant derivative ∇_α^0 does not preserve the analyticity,

$$
[D_\alpha^{++}, \nabla_\beta^0] = -\frac{1}{4}\varepsilon_{\alpha\beta}(D^{++})^2(V_L^{--} - V_R^{--}) = \varepsilon_{\alpha\beta}(W_L^{++} - W_R^{++}). \tag{3.27}
$$

However, one can check that the expression $\nabla_\alpha^0 + \theta_\alpha^{--}(W_L^{++} - W_R^{++})$ entering the transformations (3.25) does preserve analyticity,

$$
[D_\alpha^{++}, \nabla_\beta^0 + \theta_\beta^{--}(W_L^{++} - W_R^{++})] = 0\,. \tag{3.28}
$$

Now we compute the variation of the hypermultiplet action (3.15) with respect to the transformation (3.25),

$$
\begin{aligned}
\delta_\epsilon S_{\text{hyp}} &= i\int d\zeta^{(-4)}[\epsilon^{\alpha\,ab}D_\alpha^0(\bar{q}_a^+\nabla^{++}q_b^+) + 2\epsilon^{\alpha\,ab}\theta_\alpha^0(W_L^{++} - W_R^{++})\bar{q}_a^+ q_b^+] \\
&= 2i\int d\zeta^{(-4)}\epsilon^{\alpha\,ab}\theta_\alpha^0(W_L^{++} - W_R^{++})\bar{q}_a^+ q_b^+.
\end{aligned} \tag{3.29}
$$

The non-vanishing expression in the second line of (3.29) can be compensated by the following transformation of the gauge superfields,

$$\delta V_L^{++} = \delta V_R^{++} = \frac{8\pi}{k}\epsilon^{\alpha\,ab}\theta^0_\alpha \bar{q}^+_a q^+_b\,. \tag{3.30}$$

Indeed, applying the formula (3.20) for the variation of the Chern-Simons action, we find

$$\delta_\epsilon S_{\rm gauge} = -2i\int d\zeta^{(-4)}\epsilon^{\alpha\,ab}\theta^0_\alpha\bar{q}^+_a q^+_b(W_L^{++} - W_R^{++}), \tag{3.31}$$

which exactly cancels (3.29). Note that the gauge superfield $A^{++} = V_L^{++} - V_R^{++}$, which appears in the hypermultiplet action (3.18), is inert under the transformations (3.30), $\delta_\epsilon A^{++} = 0$. Thus we conclude that the total action (3.24) is invariant under the three extra supersymmetry transformations realized on the involved $\mathcal{N}$=3 superfields by the rules (3.25), (3.30).

The last issue is to show that the commutator of two consequent transformations (3.25) generates on shell x^m-translations of the superfields,

$$\begin{aligned}
[\delta_\mu\delta_\epsilon - \delta_\epsilon\delta_\mu]q^{+a} &= -\frac{1}{2}(\mu^{\alpha a}{}_b\epsilon^{\beta\,bc} - \mu^{\beta c}{}_b\epsilon^{\alpha\,ab})\left[\{\nabla^0_\alpha,\nabla^0_\beta\} - \varepsilon_{\alpha\beta}(\nabla^0)^2\right]q^+_c \\
&= \frac{i}{2}\mu^{(\alpha}_{bc}\epsilon^{\beta)\,bc}\nabla_{\alpha\beta}q^{+a} - \mu^{\alpha(a}{}_b\epsilon^{c)b}_\alpha(\nabla^0)^2 q^+_c\,.
\end{aligned} \tag{3.32}$$

Here $\nabla_{\alpha\beta}$ is a gauge covariant d=3 vector derivative which generates the "covariant" translations with the bosonic parameter $\mu^{(\alpha}_{bc}\epsilon^{\beta)\,bc}$ (it is a sum of ordinary x-translation and a field-dependent U(1) gauge transformation). The last term in (3.32) vanishes on shell. To show this, we first note that, by analyzing the harmonic differential equations, one can prove (see [43] for the details)

$$\nabla^{++}q^+_a = 0 \quad\Rightarrow\quad \nabla^{--}\nabla^{--}q^+_a = 0. \tag{3.33}$$

Next, using the analyticity of q^+_a we have

$$0 = D^{++\alpha}D^{++}_\alpha\nabla^{--}\nabla^{--}q^+_a = -8[(W_L^{++} - W_R^{++})\nabla^{--} + (W^0_L - W^0_R) - \nabla^{0\alpha}\nabla^0_\alpha]q^+_a\,, \tag{3.34}$$

where $W^0_{L,R} = \frac{1}{2}\mathcal{D}^{--}W^{++}_{L,R}$. Now we exploit the equations of motion for the gauge superfields, (3.21b), to deduce their corollaries

$$W_L^{++} - W_R^{++} = 0, \qquad W^0_L - W^0_R = 0\,. \tag{3.35}$$

Then (3.34) implies

$$\nabla^{0\alpha}\nabla^0_\alpha q^+_a = 0\,. \tag{3.36}$$

This completes the proof that on shell the commutator (3.32) of two extra $\mathcal{N}$=3 supersymmetry transformations (3.25) yields, modulo a field-dependent gauge transformation, the ordinary d=3 translation.

The U(1) $\times$ U(1) model also respects the appropriate generalization of the free case SO(6) R-symmetry (3.9). We shall postpone discussion of this symmetry until considering the gauge group U(N) $\times$ U(M) in the next Subsection. The U(1) $\times$ U(1) example follows from this more general case via an obvious reduction.

JHEP03(2009)096

3.3 The $\mathrm{U}(N) \times \mathrm{U}(M)$ theory

The crucial idea in constructing the $\mathcal{N}$=6 supersymmetric gauge theory in [5] was to consider the matter fields in the bifundamental representation of the $\mathrm{U}(N) \times \mathrm{U}(N)$ gauge group, the product of the fundamental representation of the left $\mathrm{U}(N)$ and the conjugated fundamental representation of the right $\mathrm{U}(N)$. Actually, one can consider the more general case of the gauge group $\mathrm{U}(N) \times \mathrm{U}(M)$, $N \neq M$ (see, e.g., [16, 28–30]):

$$(N, \bar{M}) : \quad (q^{+a})_A^{\underline{B}}, \qquad (\bar{N}, M) : \quad (\bar{q}_a^+)_{\underline{B}}^A, \tag{3.37}$$

where $A = 1, \ldots, N$ and $\underline{B} = 1, \ldots, M$. Hereafter, the underlined indices refer to the right $\mathrm{U}(M)$ gauge group. Yet admissible is another type of the bifundamental representation, the product of two fundamental representations [30]:

$$(N, M) : \quad (q^{+a})_{A\underline{B}}, \quad (\bar{N}, \bar{M}) : \quad (\bar{q}_a^+)^{A\underline{B}}. \tag{3.38}$$

In this Subsection we shall focus on the case (3.37) as the standard and most instructive one. The case (3.38) as well as some other options admitting hidden supersymmetries will be shortly addressed in the section 4.

The gauge superfields for the groups $\mathrm{U}(N)$ and $\mathrm{U}(M)$ are given by the antihermitian matrices $(V_L^{++})_B^A$ and $(V_R^{++})_{\underline{B}}^{\underline{A}}$. The gauge interaction of the hypermultiplets with the gauge superfields in the $(N, \bar{M})$-model under consideration reads

$$\begin{aligned}
(\nabla^{++} q^{+a})_A^{\underline{B}} &= \mathcal{D}^{++} (q^{+a})_A^{\underline{B}} + (V_L^{++})_A^B (q^{+a})_B^{\underline{B}} - (q^{+a})_A^{\underline{A}} (V_R^{++})_{\underline{A}}^{\underline{B}}, \\
(\nabla^{++} \bar{q}_a^+)_{\underline{B}}^A &= \mathcal{D}^{++} (\bar{q}_a^+)_{\underline{B}}^A - (\bar{q}_a^+)_{\underline{B}}^B (V_L^{++})_B^A + (V_R^{++})_{\underline{B}}^{\underline{A}} (\bar{q}_a^+)_{\underline{A}}^A.
\end{aligned} \tag{3.39}$$

The matrix form of the $(N, \bar{M})$ harmonic derivative is

$$\begin{aligned}
\nabla^{++} q^{+a} &= \mathcal{D}^{++} q^{+a} + V_L^{++} q^{+a} - q^{+a} V_R^{++}, \ \nabla^{++} \bar{q}_a^+ = \mathcal{D}^{++} \bar{q}_a^+ - \bar{q}_a^+ V_L^{++} + V_R^{++} \bar{q}_a^+, \\
\bar{q}_a^+ &= (q^{+a})^\dagger.
\end{aligned} \tag{3.40}$$

The superfield $\nabla^{++} q^{+a}$ transforms covariantly under the following infinitesimal gauge transformations

$$\begin{aligned}
\delta q^{+a} &= \Lambda_L q^{+a} - q^{+a} \Lambda_R, & \delta \bar{q}_a^+ &= \Lambda_R \bar{q}_a^+ - \bar{q}_a^+ \Lambda_L, \\
\delta V_L^{++} &= -\mathcal{D}^{++} \Lambda_L - [V_L^{++}, \Lambda_L], & \delta V_R^{++} &= -\mathcal{D}^{++} \Lambda_R - [V_R^{++}, \Lambda_R].
\end{aligned} \tag{3.41}$$

The analytic gauge parameters Λ_L and Λ_R are antihermitian matrices, $\Lambda_L^\dagger = -\Lambda_L$, $\Lambda_R^\dagger = -\Lambda_R$. The hermitian conjugation for the other superfields is defined as

$$(q^{+a})^\dagger = \bar{q}_a^+, \quad (\bar{q}_a^+)^\dagger = -q^{+a}, \quad (V_L^{++})^\dagger = -V_L^{++}, \quad (V_R^{++})^\dagger = -V_R^{++}. \tag{3.42}$$

After fixing the notations, we turn to the actions. We write down the non-Abelian $\mathcal{N}$=6 supersymmetric action as a direct generalization of the $\mathrm{U}(1) \times \mathrm{U}(1)$ actions (3.24), (3.14), (3.15):

$$S_{\mathcal{N}=6} = S_{\rm gauge} + S_{\rm hyp}, \tag{3.43}$$

$$S_{\rm gauge} = S_{\rm CS}[V_L^{++}] - S_{\rm CS}[V_R^{++}], \tag{3.44}$$

$$S_{\rm hyp} = \mathrm{tr} \int d\zeta^{(-4)} \bar{q}_a^+ \nabla^{++} q^{+a}, \tag{3.45}$$

where the Chern-Simons action $S_{\rm CS}[V^{++}]$ is given by (2.25). The analytic superfield equations of motion corresponding to (3.43) read

$$(\nabla^{++}q^{+a})_{B}^{\underline{A}} = (\nabla^{++}\bar{q}^{+a})_{\underline{A}}^{B} = 0\,, \tag{3.46}$$

$$(W_L^{++})_A^B = -i\frac{4\pi}{k}\,(q^{+a})_A^{\underline{D}}(\bar{q}_a^{+})_{\underline{D}}^{B}\,, \quad (W_R^{++})_{\underline{A}}^{\underline{B}} = -i\frac{4\pi}{k}\,(\bar{q}_a^{+})_{\underline{A}}^{D}(q^{+a})_{D}^{\underline{B}}\,. \tag{3.47}$$

3.3.1 $\mathcal{N}$=6 supersymmetry

We claim that the action (3.43) is invariant under the following three extra supersymmetry transformations:

$$\begin{aligned}
\delta_\epsilon q^{+a} &= i\epsilon^{\alpha\,(ab)}\hat{\nabla}^0_\alpha q_b^+\,, & \delta_\epsilon \bar{q}_a^+ &= i\epsilon^{\alpha}_{(ab)}\hat{\nabla}^0_\alpha \bar{q}^{+b}\,,\\
\delta_\epsilon V_L^{++} &= \frac{8\pi}{k}\epsilon^{\alpha\,(ab)}\theta^0_\alpha q_a^+\bar{q}_b^+\,, & \delta_\epsilon V_R^{++} &= \frac{8\pi}{k}\epsilon^{\alpha\,(ab)}\theta^0_\alpha \bar{q}_a^+ q_b^+\,,
\end{aligned} \tag{3.48}$$

where

$$\begin{aligned}
\hat{\nabla}^0_\alpha q_b^+ &= \nabla^0_\alpha q_b^+ + \theta^{--}_\alpha(W_L^{++}q_b^+ - q_b^+ W_R^{++})\,,\\
\nabla^0_\alpha q_a^+ &= D^0_\alpha q_a^+ + V^0_{L\,\alpha}q_a^+ - q_a^+ V^0_{R\,\alpha}\,, \quad V^0_{L,R\,\alpha} = -\frac{1}{2}D^{++}_\alpha V^{--}_{L,R}
\end{aligned} \tag{3.49}$$

and $\hat{\nabla}^0_\alpha \bar{q}^{+b}, \nabla^0_\alpha \bar{q}^{+b}$ are obtained via the $\widetilde{\ }$ conjugation. The modified gauge-covariant derivative $\hat{\nabla}^0_\alpha$ preserves the $\mathcal{N}$=3 analyticity as opposed to ∇^0_α.

The variation of the hypermultiplet action (3.45) under the transformations (3.48) is

$$\begin{aligned}
\delta_\epsilon S_{\rm hyp} &= 2i\,{\rm tr}\int d\zeta^{(-4)}\epsilon^{\alpha\,(ab)}\theta^0_\alpha(W_L^{++}q_a^+\bar{q}_b^+ - W_R^{++}\bar{q}_a^+ q_b^+)\\
&\quad+\frac{8\pi}{k}\,{\rm tr}\int d\zeta^{(-4)}\epsilon^{\alpha\,(ab)}\theta^0_\alpha[q_a^+\bar{q}_b^+ q^{+c}\bar{q}_c^+ + \bar{q}_a^+ q_b^+\bar{q}^{+c}q_c^+].
\end{aligned} \tag{3.50}$$

Using a simple Fierz rearrangement, one can check that

$${\rm tr}\left[q^+_{(a}\bar{q}^+_{b)}q^{+c}\bar{q}^+_c + \bar{q}^+_{(a}q^+_{b)}\bar{q}^{+c}q^+_c\right] = 0\,, \tag{3.51}$$

so the variation (3.50) is reduced to

$$\delta_\epsilon S_{\rm hyp} = 2i\,{\rm tr}\int d\zeta^{(-4)}\epsilon^{\alpha\,ab}\theta^0_\alpha(W_L^{++}q_a^+\bar{q}_b^+ - W_R^{++}\bar{q}_a^+ q_b^+). \tag{3.52}$$

This expression is exactly canceled by the variation of the Chern-Simons term (3.44),

$$\delta_\epsilon S_{\rm gauge} = -2i\,{\rm tr}\int d\zeta^{(-4)}\epsilon^{\alpha\,ab}\theta^0_\alpha(W_L^{++}q_a^+\bar{q}_b^+ - W_R^{++}\bar{q}_a^+ q_b^+). \tag{3.53}$$

As a result, we proved that the total action (3.43) is invariant under the $\mathcal{N}$=3 supersymmetry transformations (3.48),

$$\delta_\epsilon S_{\mathcal{N}=6} = \delta_\epsilon(S_{\rm gauge} + S_{\rm hyper}) = 0. \tag{3.54}$$

Together with the manifest $\mathcal{N}$=3 supersymmetries of the $\mathcal{N}$=3 superspace, the transformations (3.48) form $\mathcal{N}$=6 supersymmetry. Therefore we conclude that the action (3.43)

JHEP03(2009)096

provides the formulation of the $(N,\bar{M})$ $\mathcal{N}$=6 Chern-Simons model in the $\mathcal{N}$=3 harmonic superspace. Like in the $\mathrm{U}(1)\times\mathrm{U}(1)$ model, the extra hidden $\mathcal{N}$=3 supersymmetry, as opposed to the manifest $\mathcal{N}$=3 one, has the correct closure on d=3 translations only modulo the superfield equations of motion (3.46), (3.47) and a field-dependent gauge transformation, i.e. it is essentially on-shell.

3.3.2 SO(6) R-symmetry

A natural generalization of the transformations (3.9) to the $\mathrm{U}(N)\times\mathrm{U}(M)$ case is

$$\begin{aligned}\delta_\lambda q^{+a} &= -i[\lambda^{0(ab)} - \lambda^{++(ab)}\hat{\nabla}^{--} - 2\lambda^{--(ab)}\theta^{++\alpha}\hat{\nabla}^0_\alpha + 4\lambda^{0(ab)}\theta^{0\alpha}\hat{\nabla}^0_\alpha]q^+_b,\\ \delta_\lambda \bar{q}^+_a &= -i[\lambda^0_{(ab)} - \lambda^{++}_{(ab)}\hat{\nabla}^{--} - 2\lambda^{--}_{(ab)}\theta^{++\alpha}\hat{\nabla}^0_\alpha + 4\lambda^0_{(ab)}\theta^{0\alpha}\hat{\nabla}^0_\alpha]\bar{q}^{+b},\end{aligned}\tag{3.55}$$

where $\hat{\nabla}^{--}$ and $\hat{\nabla}^0_\alpha$ are gauge-covariant analyticity-preserving derivatives:

$$\begin{aligned}\hat{\nabla}^0_\alpha &= \nabla^0_\alpha + \theta^{--}_\alpha W^{++}, & \{D^{++}_\alpha, \hat{\nabla}^0_\beta\} &= 0,\\ \hat{\nabla}^{--} &= \nabla^{--} + 2\theta^{\alpha--}\nabla^0_\alpha + (\theta^{--})^2 W^{++}, & [D^{++}_\alpha, \hat{\nabla}^{--}] &= 0.\end{aligned}\tag{3.56}$$

The variation of the hypermultiplet action (3.45) under (3.55) is

$$\delta_\lambda S_{\rm hyp} = i\,\mathrm{tr}\int d\zeta^{(-4)}\kappa_{(ab)}\bar{q}^{+a}(W^{++}_L q^{+b} - q^{+b}W^{++}_R),\tag{3.57}$$

where

$$\kappa_{(ab)} = 4\lambda^{--}_{(ab)}(\theta^0\theta^{++}) - 8\lambda^0_{(ab)}(\theta^0)^2.\tag{3.58}$$

Here we have used the following identities

$$[\nabla^{++},\hat{\nabla}^0_\alpha]q^+_a = 2\theta^0_\alpha(W^{++}_L q^+_a - q^+_a W^{++}_R),\qquad [\nabla^{++},\hat{\nabla}^{--}]q^+_a = (1+4\theta^{0\alpha}\hat{\nabla}^0_\alpha)q^+_a.\tag{3.59}$$

To cancel the variation (3.57) we have to make the following transformation of the gauge superfields

$$\delta_\lambda V^{++}_L = \frac{4\pi}{k}\kappa^{ab}q^+_a\bar{q}^+_b,\qquad \delta_\lambda V^{++}_R = \frac{4\pi}{k}\kappa^{ab}\bar{q}^+_a q^+_b,\tag{3.60}$$

under which the Chern-Simons action (3.44) varies as

$$\delta_\lambda S_{\rm gauge} = -i\,\mathrm{tr}\int d\zeta^{(-4)}\kappa^{ab}(q^+_a\bar{q}^+_b W^{++}_L - \bar{q}^+_a q^+_b W^{++}_R).\tag{3.61}$$

The variations (3.60) performed in the hypermultiplet action produce quartic q^+ terms which cancel each other as a consequence of the same identity (3.51) as in the case of hidden $\mathcal{N}$=3 supersymmetry.

As a result, we proved that the action (3.43) is invariant under the transformations (3.55),

$$\delta_\lambda S_{\mathcal{N}=6} = 0,\tag{3.62}$$

and, therefore, respects the SO(6) R-symmetry group.

It is interesting to calculate the Lie bracket of the SO(6)/SO(4) transformations with the manifest $\mathcal{N}=3$ supersymmetry. In the analytic basis, the latter is realized by the following differential operator

$$\delta_\epsilon = \epsilon^{0\alpha}\left(\frac{\partial}{\partial\theta^{0\alpha}} + 2i\theta^{0\beta}\partial_{\beta\alpha}\right) + \epsilon^{++\alpha}\frac{\partial}{\partial\theta^{++\alpha}} + \epsilon^{--\alpha}\left(\frac{\partial}{\partial\theta^{--\alpha}} - 2i\theta^{++\beta}\partial_{\beta\alpha}\right). \tag{3.63}$$

Then

$$[\delta_\lambda\delta_\epsilon - \delta_\epsilon\delta_\lambda]\,(V^{++}_L)^A_B = \frac{8\pi}{k}\omega^\alpha_{(ab)}\,\theta^0_\alpha\,(q^{+a})^{\underline{B}}_B(\bar{q}^{+b})^A_{\underline{B}} + \frac{4\pi}{k}(D^{++}f^{--}_{(ab)})\,(q^{+a})^{\underline{B}}_B(\bar{q}^{+b})^A_{\underline{B}}, \tag{3.64}$$

$$\begin{aligned}[\delta_\lambda\delta_\epsilon - \delta_\epsilon\delta_\lambda]\,(q^{+a})^{\underline{B}}_A &= i\omega^{\alpha(ab)}\left(\hat{\nabla}^0_\alpha q^+_b\right)^{\underline{B}}_A \\ &\quad -if^{--(ab)}\left[(W^{++}_L)^D_A\,(q^+_b)^{\underline{B}}_D - (W^{++}_R)^{\underline{B}}_{\underline{C}}\,(q^+_b)^{\underline{C}}_A\right],\end{aligned} \tag{3.65}$$

where

$$\omega^\alpha_{(ab)} = 2\lambda_{(ij)(ab)}\epsilon^{(ij)\alpha} \tag{3.66}$$

and

$$f^{--(ab)} = 2\lambda^{--(ab)}(\epsilon^{--\alpha}\theta^{++}_\alpha) - 4\lambda^{0(ab)}(\epsilon^{--\alpha}\theta^0_\alpha)\,. \tag{3.67}$$

The bracket for $(V^{++}_R)^{\underline{A}}_{\underline{B}}$ has a form quite analogous to (3.64).

First terms in (3.64), (3.65) are just the transformations of the hidden $\mathcal{N}=3$ supersymmetry which extends the manifest one to $\mathcal{N}=6$. The remaining terms are reduced on shell to a field-dependent gauge transformation. Indeed, with making use of the equations of motion (3.46), (3.47) these "superfluous" terms in the bracket transformations of V^{++}_L, V^{++}_R and q^{+a} can be represented, respectively, as

$$-(\nabla^{++}\Lambda)^B_A\,, \quad -(\nabla^{++}\tilde{\Lambda})^{\underline{A}}_{\underline{B}}\,, \quad \Lambda^D_A\,(q^{+a})^{\underline{B}}_D - \tilde{\Lambda}^{\underline{B}}_{\underline{D}}(q^{+a})^{\underline{D}}_A\,, \tag{3.68}$$

where

$$\Lambda^B_A := -\frac{4\pi}{k}\,f^{--}_{(ab)}\,(q^{+a})^{\underline{C}}_A\,(\bar{q}^{+b})^B_{\underline{C}}\,, \quad \tilde{\Lambda}^{\underline{B}}_{\underline{A}} := -\frac{4\pi}{k}\,f^{--}_{(ab)}\,(q^{+a})^{\underline{B}}_C\,(\bar{q}^{+b})^C_{\underline{A}}\,. \tag{3.69}$$

Thus, the transformations of hidden supersymmetries can be equivalently derived as an essential part of the Lie bracket of the explicit $\mathcal{N}$=3 supersymmetry with the hidden internal automorphisms transformations (i.e. the part which retains on shell and is not reduced to a gauge transformation).

3.4 Scalar potential

One of the basic features of the ABJM model is the sextic potential of the scalar fields. In [5] it was presented in the manifestly SU(4) invariant form. Following the ABJM terminology, there are four complex scalars, two of which, A_1 and A_2, are in the bifundamental representation while the other two, B_1 and B_2, are in the anti-bifundamental representation. These scalars are combined into the SU(4) spinors ("quark" and "anti-quark"):

$$C_I = (A_1, A_2, B_1^\dagger, B_2^\dagger), \qquad C^{\dagger I} = (A_1^\dagger, A_2^\dagger, B_1, B_2). \tag{3.70}$$

In terms of these quantities the scalar potential is written as

$$V_{(ABJM)} = \frac{4\pi^2}{k^2}\mathrm{tr}\left(\frac{1}{3}\mathrm{tr}\,(C_I C^{\dagger I} C_J C^{\dagger J} C_K C^{\dagger K} + \frac{1}{3} C_I C^{\dagger J} C_J C^{\dagger K} C_K C^{\dagger I}\right.$$
$$\left. -2 C_I C^{\dagger I} C_J C^{\dagger K} C_K C^{\dagger J} + \frac{4}{3} C_I C^{\dagger K} C_J C^{\dagger I} C_K C^{\dagger J}\right). \qquad (3.71)$$

In our $\mathcal{N}$=3 harmonic superspace formulation of the ABJM model the action (3.43) contains no explicit superfield potential, it involves only minimal gauge interactions of the hypermultiplets with the gauge superfields. As was already mentioned, such a form of the action is uniquely prescribed by $\mathcal{N}$=3 superconformal invariance.

Here we demonstrate that the scalar potential (3.71) naturally stems from the superfield action (3.43) as a result of elimination of auxiliary fields.

Upon reducing the action (3.43) to the component form, the contributions to the scalar potential come from both the hypermultiplet and Chern-Simons actions (3.44), (3.45). To derive the scalar potential, we take $V^{++}_{L,R}$ in the Wess-Zumino gauge (2.20) and discard there gauge fields and all fermionic fields. Further, based on the dimensionality reasoning, we single out those auxiliary bosonic fields in the hypermultiplet superfields which are relevant to forming the on-shell scalar potential. As a result we find that it suffices to leave only the following component fields:

$$\begin{aligned} V^{++}_{L,R} &= 3(\theta^{++})^2 u^-_k u^-_l \phi^{kl}_{L,R} + 3i(\theta^{++})^2(\theta^0)^2 u^-_k u^-_l X^{kl}_{L,R} + \cdots\,, \\ q^{+a} &= u^+_i f^{i\,a} + (\theta^0)^2 g^{i\,a} u^+_i + (\theta^{++}\theta^0) h^{i\,a} u^-_i + \cdots\,, \\ \bar q^+_a &= -u^+_i \bar f^i_a + (\theta^0)^2 \bar g^i_a u^+_i + (\theta^{++}\theta^0)\bar h^i_a u^-_i + \cdots\,. \end{aligned} \qquad (3.72)$$

Now, using the component structure of the Chern-Simons action (2.32) we can explicitly write down those terms in the action (3.44) which are responsible for the scalar potential,

$$\begin{aligned} S_{\rm gauge} &= -\frac{ik}{6\pi}\mathrm{tr}\int d^3x(\phi^m_{Lk}[\phi^n_{Lm},\phi^k_{Ln}] - \phi^m_{Rk}[\phi^n_{Rm},\phi^k_{Rn}]) \\ &\quad + \frac{k}{4\pi}\mathrm{tr}\int d^3x(\phi^{ij}_L X_{Lij} - \phi^{ij}_R X_{Rij}) + \cdots\,. \end{aligned} \qquad (3.73)$$

To find the component structure of the appropriate part of the hypermultiplet action we eliminate the auxiliary fields $g^{i\,a}$, $h^{i\,a}$ from the equation of motion $\nabla^{++}q^{+a} = 0$,

$$h^{i\,a} = -2g^{i\,a} = 2\phi^{ij}_L f^a_j - 2 f^a_j \phi^{ij}_R \qquad (3.74)$$

and substitute the component expansions (3.72) into the hypermultiplet action (3.45). After integration over the Grassmann and harmonic variables we obtain

$$\begin{aligned} S_{\rm hyp} = \mathrm{tr}\int d^3x[&-\bar f_{i\,a}\phi^{ij}_L \phi_{Ljk} f^{k\,a} + \bar f_{i\,a}\phi^{ij}_L f^{k\,a}\phi_{Rjk} + \bar f_{i\,a}\phi_{Ljk} f^{k\,a}\phi^{ij}_R - \bar f_{i\,a} f^{k\,a}\phi_{Rjk}\phi^{ij}_R \\ &- i\bar f_{i\,a} X^{ij}_L f^a_j + i\bar f_{i\,a} f^a_j X^{ij}_R + \cdots]. \end{aligned} \qquad (3.75)$$

The auxiliary fields $X_{L,R}$ appear in the action $S_{\rm gauge} + S_{\rm hyp}$ as Lagrange multipliers for the relations

$$\phi^{ij}_L = \frac{2\pi i}{k}(f^{i\,a}\bar f^j_a + f^{j\,a}\bar f^i_a), \qquad \phi^{ij}_R = -\frac{2\pi i}{k}(\bar f^{i\,a} f^j_a + \bar f^{j\,a} f^i_a). \qquad (3.76)$$

JHEP03(2009)096

As the final step, we substitute these expressions for the auxiliary fields back into the actions (3.73), (3.75) and, after some simple algebra, obtain the scalar potential in the following form

$$\begin{aligned}V_{\rm scalar} = & -\frac{8\pi^2}{3k^2}{\rm tr}\,\{f^{i\,a}\bar f_{k\,a}(f^{j\,b}\bar f_{i\,b}+f^b_i\bar f^j_b)(f^{k\,c}\bar f_{j\,c}+f^c_j\bar f^k_c)\\ & +\bar f^{i\,a}f_{k\,a}(\bar f^{j\,b}f_{i\,b}+\bar f^b_i f^j_b)(\bar f^{k\,c}f_{j\,c}+\bar f^c_j f^k_c)\}\\ & -\frac{4\pi^2}{3k^2}{\rm tr}\,\{f^{i\,a}\bar f_{k\,a}(f^{k\,c}\bar f_{j\,c}+f^c_j\bar f^k_c)(f^{j\,b}\bar f_{i\,b}+f^b_i\bar f^j_b)\\ & +\bar f^{i\,a}f_{k\,a}(\bar f^{k\,c}f_{j\,c}+\bar f^c_j f^k_c)(\bar f^{j\,b}f_{i\,b}+\bar f^b_i f^j_b)\}\\ & -\frac{4\pi^2}{k^2}{\rm tr}\,\{\bar f^a_i(f^{i\,c}\bar f^j_c+f^{j\,c}\bar f^i_c)f^k_a(\bar f^b_j f_{k\,b}+\bar f^b_k f_{j\,b})\\ & +\bar f^a_i(f^c_j\bar f_{k\,c}+f^c_k\bar f_{j\,c})f^k_a(\bar f^{i\,b}f^j_b+\bar f^{j\,b}f^i_b)\}. \end{aligned} \tag{3.77}$$

The potential (3.77) looks rather complicated and its identity with (3.71) is not immediately obvious. To show the coincidence of these two expressions, we pass to the ABJM notations (3.70) by identifying

$$\begin{aligned} f^{i\,a} &= (f^{11},f^{12},f^{21},f^{22})=(A_1,A_2,B_1^\dagger,B_2^\dagger)=C_I,\\ \bar f_{i\,a} &= (\bar f_{11},\bar f_{12},\bar f_{21},\bar f_{22})=(A_1^\dagger,A_2^\dagger,B_1,B_2)=C^{\dagger I}. \end{aligned} \tag{3.78}$$

Substituting (3.78) into (3.77) and making appropriate Fierz rearrangements, after some tedious computation we end up with the desired identity

$$V_{\rm scalar}=V_{(ABJM)}. \tag{3.79}$$

Thus we have explicitly shown that the scalar potential derived from the superfield action (3.43) is just the potential found by ABJM [5]. We point out once more that in our $\mathcal{N}$=3 superfield formulation the scalar potential emerges solely as a result of elimination of auxiliary fields, without any presupposed superfield potential. The other interaction terms in the ABJM model (e.g., the quartic interaction of two scalars with two fermions, etc) originate from (3.43) in a similar way.

4 Other options

Here we discuss some other choices of the gauge group and/or of the representation of the hypermultiplet superfields admitting additional hidden supersymmetries and R-symmetries.

4.1 The (N,M) model

The $\mathcal{N}$=6 supersymmetry and SO(6) R-symmetry in the (N,M) model corresponding to the choice (3.38) can be proved following the same line as in the case of $(N,\bar M)$ model. The hypermultiplet action is

$$S'_{\rm hyp}=\int d\zeta^{(-4)}(\bar q^+_a)^{A\underline{A}}\nabla^{++}(q^{+a})_{A\underline{A}}\,, \tag{4.1}$$

where

$$(\nabla^{++}q^{+a})_{A\underline{A}} = \mathcal{D}^{++}(q^{+a})_{A\underline{A}} + (V^{++}_L)^B_A(q^{+a})_{B\underline{A}} + (V^{++}_R)^{\underline{B}}_{\underline{A}}(q^{+a})_{A\underline{B}}\,. \tag{4.2}$$

The additional three supersymmetry transformations are

$$\delta_\epsilon(q^{+a})_{A\underline{A}} = i\epsilon^{(ab)\alpha}\hat{\nabla}^0_\alpha(q^+_b)_{A\underline{A}}\,, \tag{4.3}$$

$$\delta_\epsilon(V^{++}_L)^B_A = \frac{8\pi}{k}\epsilon^{\alpha\,(ab)}\theta^0_\alpha(q^+_a)_{A\underline{B}}(\bar{q}^+_b)^{B\underline{B}}\,, \quad \delta_\epsilon(V^{++}_R)^{\underline{B}}_{\underline{A}} = -\frac{8\pi}{k}\epsilon^\alpha_{(ab)}\theta^0_\alpha(q^{+a})_{B\underline{A}}(\bar{q}^{+b})^{B\underline{B}}\,, \tag{4.4}$$

where now

$$\begin{aligned}\hat{\nabla}^0_\alpha(q^+_b)_{A\underline{A}} = D^0_\alpha(q^+_b)_{A\underline{A}} &+ [(V^0_{L\alpha})^B_A + \theta^{--}_\alpha(W^{++}_L)^B_A](q^+_b)_{B\underline{A}} \\ &+ [(V^0_{R\alpha})^{\underline{B}}_{\underline{A}} + \theta^{--}_\alpha(W^{++}_R)^{\underline{B}}_{\underline{A}}](q^+_b)_{A\underline{B}}\}\,.\end{aligned} \tag{4.5}$$

The transformations of the hidden SO(6)/[SU(2)$_R$ $\times$ SU(2)$_{\rm ext}$] R-symmetry mimic eqs. (3.55)–(3.60), the only difference consists in that the variation $\delta_\lambda V^{++}_R$ has the opposite sign as compared to $\delta_\lambda V^{++}_R$, like in (4.4). The invariance of the total gauge-hypermultiplet action is checked as in the previously considered $(N,\bar{M})$ model. The cancelation of the quartic terms in the full variations of the hypermultiplet action is a consequence of the identity similar to (3.51).

4.2 SU(N) $\times$ SU(N) model

Let us come back to the hypermultiplet superfield $(N,\bar{M})$ model (3.37), choose there $N = M$ and restrict the gauge group to be SU(N) $\times$ SU(N). The hidden $\mathcal{N}$=6 supersymmetry transformations (3.48), as well as the SO(6) transformations (3.60), should be slightly modified in this case in order to obey the tracelessness restrictions of the gauge group. In particular, eqs. (3.48) change as

$$\begin{aligned}\delta_\epsilon V^{++}_L &= \frac{8\pi}{k}\epsilon^{\alpha(ab)}\theta^0_\alpha\left(q^+_a\bar{q}^+_b - \frac{1}{N}{\rm tr}\, q^+_a\bar{q}^+_b\right), \\ \delta_\epsilon V^{++}_R &= \frac{8\pi}{k}\epsilon^{\alpha(ab)}\theta^0_\alpha\left(\bar{q}^+_a q^+_b - \frac{1}{N}{\rm tr}\, q^+_a\bar{q}^+_b\right)\end{aligned} \tag{4.6}$$

(equations of motion for V^{++}_L, V^{++}_R (3.47) undergo a similar modification). Analogous tracelessness conditions should be imposed on the hidden $\mathcal{N}$=6 supersymmetry and SO(6) R-symmetry transformations of the (N,M) model (3.38) restricted to $N = M$ and to the gauge group SU(N) $\times$ SU(N).

In both cases the total gauge - hypermultiplet actions remain invariant because the unwanted quartic terms in the variations of the hypermultiplet actions induced by δV^{++}_L and δV^{++}_R vanish as in the original U(N)$\times$U(M) settings. Note that the restriction of two U(1) factors in U(N)$\times$U(M) to the diagonal U(1) does not break the hidden $\mathcal{N}$=6 supersymmetry and SO(6) R-symmetry for both the (N,M) and $(N,\bar{M})$ models since the trace parts in the variations (3.48), (3.60) and (4.4) coincide and cancel each other in the appropriate variations of the hypermultiplet action. As a result, the gauge group SU(N)$\times$SU(M)$\times$U(1) is the admissible option for the existence of $\mathcal{N}$=6 supersymmetry and SO(6) R-symmetry, in agreement with the conclusion made in [28]. On the contrary, when restricting the gauge group to $\mathrm{SU}(N)\times\mathrm{SU}(M)$, $N\neq M$, there survive quartic terms $\sim(1/N-1/M)$ in these variations and there is no way to cancel them. Thus for the gauge group $\mathrm{SU}(N)\times\mathrm{SU}(M)$ both models have neither hidden supersymmetry nor hidden R-symmetry.

The $(N,\bar N)$ model for the gauge group $\mathrm{SU}(N)\times\mathrm{SU}(N)$ is invariant under the P-parity transformations

$$\begin{aligned} PV_L^{++}(\zeta_P) &= V_R^{++}(\zeta), \quad PW_L^{++}(\zeta_P) = -W_R^{++}, \\ P(q^{+a})^{\underline{B}}_A(\zeta_P) &= (\bar q^{+a})^B_{\underline{A}}(\zeta). \end{aligned} \tag{4.7}$$

The (N,N) model for the group $\mathrm{SU}(N)\times\mathrm{SU}(N)$ also respects P-parity which, on the hypermultiplets, is represented by the following transformations

$$P(q^{+a})_{A\underline{B}}(\zeta_P) = (q^{+a})_{B\underline{A}}(\zeta)\,, \quad P(\bar q^{+}_a)^{A\underline{B}}(\zeta_P) = (\bar q^{+}_a)^{B\underline{A}}(\zeta)\,. \tag{4.8}$$

4.3 $\mathbf{O}(N)\times\mathbf{USp}(2M)$ model

At the component and $\mathcal{N}$=2 superfield level, this interesting option was proposed in [16, 29, 30]. Here we treat it within the $\mathcal{N}$=3 harmonic superfield formalism.

Let us consider the hypermultiplet matrix superfield

$$(q^{+a})^{\underline{A}}_A, \tag{4.9}$$

where $A=1,\ldots,N$ is the vector index of the real $\mathrm{SO}(N)$ group and $\underline{A}=1,\ldots,2M$ is the spinor index of the $\mathrm{USp}(2M)$ group which is defined as a subgroup in $\mathrm{U}(2M)$ such that it preserves the skew-symmetric metric

$$\Omega_{\underline{AB}}, \quad \Omega^{\underline{AB}} = -\overline{(\Omega_{\underline{AB}})}\,, \quad \Omega_{\underline{AB}}\Omega^{\underline{BC}} = \delta^{\underline{C}}_{\underline{A}}\,. \tag{4.10}$$

This metric, like ε_{ik} in the $\mathrm{SU}(2)=\mathrm{USp}(2)$ case, can be used to raise or lower the fundamental representation indices $\underline{A}$. The index $a=1,2$ is treated as the global $\mathrm{SO}(2)$ one in this case. The corresponding $\tilde{}$ conjugation rules for hypermultiplets are

$$[\widetilde{(q^{+a})^{\underline{A}}_A}] = \Omega_{\underline{AB}}\,(q^{+a})^{\underline{B}}_A\,. \tag{4.11}$$

Taking into account the definitions (4.10), this pseudoreality condition is compatible with the property that the $\tilde{}$ conjugation for the q^+ superfields squares to -1.[6] The gauge-covariantized harmonic derivative is defined as

$$\begin{aligned} \nabla^{++}(q^{+a})^{\underline{A}}_A &= \mathcal{D}^{++}(q^{+a})^{\underline{A}}_A + (V_L^{++})_{AB}(q^{+a})^{\underline{A}}_B - (V_R^{++})^{\underline{A}}_{\underline{B}}(q^{+a})^{\underline{B}}_A\,, \\ (V_L^{++})_{AB} &= -(V_L^{++})_{BA}\,, \quad \Omega_{\underline{DA}}\,(V_R^{++})^{\underline{A}}_{\underline{B}} = \Omega_{\underline{BA}}\,(V_R^{++})^{\underline{A}}_{\underline{D}}\,. \end{aligned} \tag{4.12}$$

This gauge group assignment of the hypermultiplet superfields is compatible with only two additional supersymmetry transformations

$$\delta_\epsilon(q^{+a})^{\underline{A}}_A = \epsilon^{\alpha\,(ab)}\hat\nabla^0_\alpha(q^{+b})^{\underline{A}}_A\,, \tag{4.13}$$

[6] Giving up the pseudoreality condition amounts to a non-minimal variant with two copies of the pseudoreal q^{+a}. This enlargement of the field representation ($16NM$ real physical scalar as compared to $8NM$ in the case with the pseudoreality condition) does not introduce any new feature.

JHEP03(2009)096

where

$$\epsilon^{\alpha\,(ab)} = \epsilon_1^\alpha(\tau_1)^{ab} + \epsilon_3^\alpha(\tau_3)^{ab}\,, \tag{4.14}$$

ϵ_1^α and ϵ_3^α being real spinors (i.e. $\epsilon^{\alpha\,(ab)}$ is the rank 2 symmetric traceless SO(2) tensor), τ_1 and τ_3 are Pauli matrices. The two additional transformations of the $\mathrm{SO}(N)\times\mathrm{USp}(2M)$ prepotentials have the form

$$\delta_\epsilon(V_L^{++})_{AB} = -\frac{8i\pi}{k}\epsilon^{\alpha(ab)}(q^{+a})^{\underline{B}}_A(q^{+b})_{B\underline{B}}, \quad \delta_\epsilon(V_R^{++})^{\underline{B}}_{\underline{A}} = -\frac{8i\pi}{k}\epsilon^{\alpha(ab)}(q^{+a})^{\underline{B}}_B(q^{+b})_{B\underline{A}}. \tag{4.15}$$

The total $\mathrm{SO}(N)\times\mathrm{USp}(2M)$ Chern-Simons-hypermultiplet action

$$S = S_{\mathrm{CS}}(V_L^{++}) - S_{\mathrm{CS}}(V_R^{++}) + \int d\zeta^{(-4)}\, q^{+a}_{A\underline{A}}\nabla^{++}q^{+a\underline{A}}_A \tag{4.16}$$

is invariant under full $\mathcal{N}$=5 supersymmetry involving the manifest off-shell $\mathcal{N}$=3 supersymmetry and the above two additional on-shell ones.

The action (4.16) is also invariant under the following hidden R-symmetry transformation

$$\delta_\lambda q^{+a\underline{A}}_A = [\lambda^{0(ab)} - \lambda^{++(ab)}\hat{\nabla}^{--} - 2\lambda^{--(ab)}\theta^{++\alpha}\hat{\nabla}^0_\alpha + 4\lambda^{0(ab)}\theta^{0\alpha}\hat{\nabla}^0_\alpha]q^{+b\underline{A}}_A\,, \tag{4.17}$$

$$\delta_\lambda(V_L^{++})_{AB} = \frac{4i\pi}{k}\varphi^{(ab)}(q^{+a})^{\underline{B}}_A(q^{+b})_{B\underline{B}},\ \delta_\lambda(V_R^{++})^{\underline{B}}_{\underline{A}} = \frac{4i\pi}{k}\varphi^{(ab)}(q^{+a})^{\underline{B}}_B(q^{+b})_{B\underline{A}}\,, \tag{4.18}$$

where

$$\varphi^{(ab)} = 4\lambda^{--(ab)}(\theta^{++\alpha}\theta^0_\alpha) - 8\lambda^{0(ab)}(\theta^0)^2 \tag{4.19}$$

and the SO(2) index (ab) refers to the rank 2 symmetric traceless SO(2) tensor. These transformations, modulo equations of motion for auxiliary fields and field-dependent gauge transformations, together with those of the groups $\mathrm{SU}(2)_c$ and SO(2), form the 10-parameter SO(5) R-symmetry (3 parameters of $\mathrm{SU}(2)_c$ plus 1 parameter of SO(2) plus 6 real parameters $\lambda^{(ik)(ab)}$ of (4.17), (4.18)). The commutator of (4.18) with the explicit $\mathcal{N}$=3 supersymmetry (3.63) yields (once again, on-shell and up to a field-dependent gauge transformation) just the hidden $\mathcal{N}$=3 supersymmetry (4.13), (4.15). The reason why the parameters $\epsilon^{\alpha(ab)}$ and $\lambda^{(ik)(ab)}$ should be symmetric in the SO(2) indices a, b is the requirement that the variation of the hypermultiplet part of the action (4.16) with respect to (4.17) can be compensated, modulo a total derivative, by the appropriate variation of the Chern-Simons actions. The further restriction that these parameters are traceless in a, b arises as the condition of vanishing of the unwanted quartic terms in the full group variation of the hypermultiplet action. After the appropriate Fierz rearrangement, these terms (with the infinitesimal transformation parameters $\epsilon^{\alpha(ab)}$ or $\lambda^{(ik)(ab)}$ detached) are gathered into the structure

$$(\epsilon^{cg}\, q^{+c}_{B\underline{A}}q^{+g}_{A\underline{D}})\,(\epsilon^{d(a}\, q^{+b)\underline{D}}_B q^{+d\underline{A}}_A)\,. \tag{4.20}$$

It is easy to check that the (ab) trace part of this expression is not vanishing and cannot be canceled with any modification of the (super)group transformations, while the traceless part is identically zero. So the transformation parameters should be symmetric traceless.

There exist some other choices of the gauge groups and/or the representation assignments of the hypermultiplet matter which seemingly admit extra supersymmetries and

R-symmetries (see e.g. [30]). We are planning to discuss them elsewhere. The cancellation of the quartic terms in the variation of the $\mathcal{N}$=3 superfield hypermultiplet action seems to be a simple powerful criterion for selecting all non-trivial possibilities.

The component forms of the Chern-Simons - hypermultiplet superfield actions considered in this section, in particular, the corresponding sextic scalar potentials, can be derived in the same way as for the U(N) × U(M) model in Subsection 3.4.

5 Models with $\mathcal{N}$=8 supersymmetry

As claimed in [5], in the case of SU(2) × SU(2) gauge group the ABJM model has an enhanced $\mathcal{N}$=8 supersymmetry and coincides with the SO(4) BLG model [2, 3]. Here we show this using the $\mathcal{N}$=3 superfield formalism.

We start from the particular SU(2) × SU(2) case of the general U(N) × U(M) action (3.43):

$$S_{su(2)} = S_{\rm CS}[V^{++}_L] - S_{\rm CS}[V^{++}_R] - \int d\zeta^{(-4)} \bar{q}^{+A\underline{A}}_a \nabla^{++} q^{+a}_{A\underline{A}} \,, \quad \widetilde{(q^{+a}_{A\underline{A}})} = -\bar{q}^{+A\underline{A}}_a \,, \tag{5.1}$$

where we have written down the doublet indices of both gauge SU(2) groups on the same level, using the equivalency of the fundamental representation of SU(2) and its conjugate. In this notation, the covariant derivative $\nabla^{++} q^{+a}_{A\underline{A}}$ is written as

$$\nabla^{++} q^{+a}_{A\underline{A}} = \mathcal{D}^{++} q^{+a}_{A\underline{A}} + (V^{++}_L)^B_A q^{+a}_{B\underline{A}} + (V^{++}_R)^{\underline{B}}_{\underline{A}} q^{+a}_{A\underline{B}} \,. \tag{5.2}$$

Now we give up the notation in which SU(2)$_{\rm ext}$ symmetry acting on the index a is manifest and will treat the superfields $q^{+1}_{A\underline{B}}$ and $q^{+2}_{A\underline{B}}$ separately. Either these superfields, together with their $\widetilde{\ }$ conjugates, can be combined into two independent pseudo-real doublets of two Pauli-Gürsey SU(2) groups [43]:

$$\begin{aligned}
&{\rm SU}(2)_{PG\,I}: && q^{+\hat{a}}_{A\underline{B}} := (q^{+1}_{A\underline{B}},\ \bar{q}^{+}_{1\,A\underline{B}}) \,, && \widetilde{(q^{+\hat{a}}_{A\underline{B}})} = -q^{+A\underline{B}}_{\hat{a}} \,, \\
&{\rm SU}(2)_{PG\,II}: && q^{+\check{a}}_{A\underline{B}} := (q^{+2}_{A\underline{B}},\ \bar{q}^{+}_{2\,A\underline{B}}) \,, && \widetilde{(q^{+\check{a}}_{A\underline{B}})} = -q^{+A\underline{B}}_{\check{a}} \,.
\end{aligned} \tag{5.3}$$

In this new notation the action (5.1) is rewritten as

$$S_{su(2)} = S_{\rm CS}[V^{++}_L] - S_{\rm CS}[V^{++}_R] - \frac{1}{2}\int d\zeta^{(-4)} \left(\mathcal{L}_I + \mathcal{L}_{\rm II}\right), \tag{5.4}$$

$$\mathcal{L}_I = q^{+A\underline{B}}_{\hat{a}} \nabla^{++} q^{+\hat{a}}_{A\underline{B}} \,, \qquad \mathcal{L}_{\rm II} = q^{+A\underline{B}}_{\check{a}} \nabla^{++} q^{+\check{a}}_{A\underline{B}} \,. \tag{5.5}$$

The covariant derivative ∇^{++} acts in the same way as in (5.2).

The rearranged action (5.4) manifests three mutually commuting off-shell SU(2) symmetry: two Pauli-Gürsey symmetries SU(2)$_{\rm PGI}$ and SU(2)$_{\rm PGII}$ realized on the hypermultiplet doublet indices $\hat{a}$ and $\check{a}$, as well as the standard automorphism SU(2)$_R$ symmetry (or SU(2)$_c$ symmetry which is indistinguishable from SU(2)$_R$ on physical fields). The original SU(2)$_{\rm ext}$ symmetry is of course also there, but in the new formulation it is realized in some implicit way. The gauge group SU(2) × SU(2) commutes with all these symmetries. The possibility to pass to two independent pseudo-real hypermultiplet superfields is directly related to the fact that the original complex hypermultiplet superfields and their conjugates prove to be in the same bifundamental representation of $\mathrm{SU}(2)\times\mathrm{SU}(2)$ because of the equivalency of the fundamental representation and its conjugate in the SU(2) case. Just due to this property one can combine them into the $\mathrm{SU}(2)_{\mathrm{PG}}$ doublets as in (5.3). As we shall see soon, this possibility is crucial for the existence of the hidden $\mathcal{N}$=8 supersymmetry and SO(8) R-symmetry in the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ model. In the case of the gauge supergroup $\mathrm{SU}(N)\times\mathrm{SU}(N), N\geq 3$, the hypermultiplets and their conjugates transform according to non-equivalent representations and therefore cannot be joined into $\mathrm{SU}(2)_{\mathrm{PG}}$ doublets (neither for the $(N,\bar{N})$ model nor for the (N,N) one). Correspondingly, their $\mathcal{N}$=6 supersymmetry and SO(6) R-symmetry are not further enhanced. The same is true for $\mathrm{U}(N)\times\mathrm{U}(M)$ models including $\mathrm{U}(2)\times\mathrm{U}(2)$ and $\mathrm{U}(1)\times\mathrm{U}(1)$ ones. We shall see that there exists an extended version of the $\mathrm{U}(1)\times\mathrm{U}(1)$ model which still admits $\mathrm{SU}(2)_{\mathrm{PG}}$ doublet structure (it involves 8 complex physical scalar fields instead of 4 such fields in the minimal $\mathrm{U}(1)\times\mathrm{U}(1)$ case). It is obtained as a reduction of the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ model and also possesses $\mathcal{N}=8$ supersymmetry and SO(8) R-symmetry. The formulation in terms of two pseudo-real hypermultiplets exists as well in the more general case of gauge groups $\mathrm{USp}(2N)\times\mathrm{USp}(2M)$ for which the bifundamental representation $(2N,2M)$ is also equivalent to its complex conjugate due to the existence of the invariant skew-symmetric metrics. For generic values of N and M, however, no hidden supersymmetries or full SO(8) R-symmetry arise in the $\mathrm{USp}(2N)\times\mathrm{USp}(2M)$ models as we argue below.

In revealing the hidden symmetries inherent in the action (5.4) we start with the R-symmetries. The most evident extra symmetry is realized by linear transformations

$$\delta_\lambda q^{+\hat a}_{A\underline{A}} = \lambda^{\hat a\check a} q^{+}_{\check a\,A\underline{A}}\,,\quad \delta_\lambda q^{+\check a}_{A\underline{A}} = \lambda^{\hat a\check a} q^{+}_{\hat a\,A\underline{A}}\,,\quad \delta_\lambda V^{++}_L = \delta_\lambda V^{++}_R = 0\,, \tag{5.6}$$

where $\lambda^{\hat a\check a}$ are four real parameters. They commute with the manifest $\mathcal{N}$=3 supersymmetry and close off shell on the product $\mathrm{SU}(2)_{\mathrm{PGI}}\times\mathrm{SU}(2)_{\mathrm{PGII}} = \mathrm{SO}(4)_{\mathrm{PG}}$. Together with the latter they generate SO(5) symmetry which is the maximal subsymmetry of the full R-symmetry group of the model under consideration which commutes with the manifest $\mathcal{N}$=3 supersymmetry.

Two other sets of the hidden internal symmetries are represented by the transformations of the form which we already met in the examples considered earlier.

The first set of additional transformations leaving the action (5.4) invariant is as follows

$$\begin{aligned}
\delta_\omega q^{+\hat a}_{A\underline{A}} &= [\omega^0 - \omega^{++}\hat\nabla^{--} - 2\omega^{--}\theta^{++\alpha}\hat\nabla^0_\alpha + 4\omega^0\theta^{0\alpha}\hat\nabla^0_\alpha] q^{+\hat a}_{A\underline{A}}\,,\\
\delta_\omega q^{+\check a}_{A\underline{A}} &= -[\omega^0 - \omega^{++}\hat\nabla^{--} - 2\omega^{--}\theta^{++\alpha}\hat\nabla^0_\alpha + 4\omega^0\theta^{0\alpha}\hat\nabla^0_\alpha] q^{+\check a}_{A\underline{A}}\,,\\
\delta_\omega (V^{++}_L)^A_B &= \frac{2i\pi}{k}\varphi\left(q^{+\check a}_{B\underline{A}} q^{+A\underline{A}}_{\hat a} - q^{+\hat a}_{B\underline{A}} q^{+A\underline{A}}_{\check a}\right),\\
\delta_\omega (V^{++}_R)^{\underline{A}}_{\underline{B}} &= \frac{2i\pi}{k}\varphi\left(q^{+\hat a}_{A\underline{B}} q^{+A\underline{A}}_{\hat a} - q^{+\check a}_{A\underline{B}} q^{+A\underline{A}}_{\check a}\right),
\end{aligned} \tag{5.7}$$

where

$$\varphi = 4\omega^{--}(\theta^{++\alpha}\theta^0_\alpha) - 8\omega^0(\theta^0)^2 \tag{5.8}$$

JHEP03(2009)096

and $\omega^0=\omega^{(ik)}u^+_iu^-_k\,, \omega^{\pm\pm}=\omega^{(ik)}u^\pm_iu^\pm_k$. The cancelation of the quartic terms in the variation of the hypermultiplet action comes about under the two conditions of the same type

$$(q^{+\hat{b}}_{A\underline{B}}\,q^{+}_{\hat{b}\,B\underline{A}})\,q^{+B\underline{B}}_{\hat{a}}=0\,,\quad (q^{+\check{b}}_{A\underline{B}}\,q^{+}_{\check{b}\,B\underline{A}})\,q^{+B\underline{B}}_{\check{a}}=0\,, \tag{5.9}$$

which are easily checked to be satisfied for the SU(2) case. These transformations in their on-shell closure yield the conformal R-symmetry group $\mathrm{SU}(2)_c$. Taken together with the $\mathrm{SU}(2)_c$ transformations, they amount to two independent SU(2) rotations of the physical fields in $q^{+\hat{b}}=f^{i\hat{b}}u^+_i+\cdots$ and $q^{+\check{b}}=f^{i\check{b}}u^+_i+\cdots$ with respect to their harmonic indices i.

The last set of hidden R-symmetry transformations is given by

$$\begin{aligned}
\delta_\sigma q^{+\hat{a}}_{A\underline{A}} &= [\sigma^{0\,\hat{a}\check{b}}-\sigma^{++\,\hat{a}\check{b}}\hat{\nabla}^{--}-2\sigma^{--\,\hat{a}\check{b}}\theta^{++\alpha}\hat{\nabla}^0_\alpha+4\sigma^{0\,\hat{a}\check{b}}\theta^{0\alpha}\hat{\nabla}^0_{\alpha]}\,q^+_{\check{b}\,A\underline{A}}\,,\\
\delta_\sigma q^{+\check{a}}_{A\underline{A}} &= -[\sigma^{0\,\hat{a}\check{b}}-\sigma^{++\,\hat{a}\check{b}}\hat{\nabla}^{--}-2\sigma^{--\,\hat{a}\check{b}}\theta^{++\alpha}\hat{\nabla}^0_\alpha+4\sigma^{0\,\hat{a}\check{b}}\theta^{0\alpha}\hat{\nabla}^0_{\alpha]}\,q^+_{\hat{b}\,A\underline{A}}\,,\\
\delta_\sigma(V^{++}_L)_{AB} &= -\frac{4i\pi}{k}\varphi^{\hat{a}\check{b}}\,q^{+\underline{B}}_{\hat{a}(A}\,q^+_{\check{b}\,B)\underline{B}}\,,\ \delta_\sigma(V^{++}_R)_{\underline{AB}}=\frac{4i\pi}{k}\varphi^{\hat{a}\check{b}}\,q^{+B}_{\hat{a}(\underline{A}}\,q^+_{\check{b}B\underline{B})}\,,
\end{aligned} \tag{5.10}$$

with

$$\varphi^{\hat{a}\check{b}}=4\sigma^{--\,\hat{a}\check{b}}(\theta^{++\alpha}\theta^0_\alpha)-8\sigma^{0\,\hat{a}\check{b}}(\theta^0)^2 \tag{5.11}$$

and $\sigma^{++\hat{a}\check{b}}=\sigma^{(ik)\hat{a}\check{b}}u^+_iu^+_k$, etc. The conditions of vanishing of the relevant quartic terms in the variation of hypermultiplet action are again (5.9).

The total number of parameters of all R-symmetries of the action (5.4) is a sum of 12 parameters of four commuting SU(2) symmetries including (5.7), of 4 parameters of the transformations (5.6) and of 12 parameters of the transformations (5.10), i.e. total of 28 parameters, the dimension of the group SO(8). Indeed, one can check that all these R-symmetry transformations close modulo field-dependent gauge transformations and superfield equations of motion, and their closure is just the maximal R-symmetry group SO(8).

Commuting (5.7) and (5.10) with the transformations of the manifest $\mathcal{N}$=3 supersymmetry, we find 5 new hidden supersymmetries, with the Lie bracket parameters $\varepsilon_\alpha\propto\omega^{(ik)}\epsilon_{\alpha(ik)}$ and $\varepsilon^{\hat{a}\check{b}}_\alpha\propto\sigma^{(ik)\,\hat{a}\check{b}}\epsilon_{\alpha(ik)}$. They are realized by the following transformations

$$\begin{aligned}
\delta_\varepsilon q^{+\hat{a}}_{A\underline{B}} &= \varepsilon^\alpha\hat{\nabla}^0_\alpha q^{+\hat{a}}_{A\underline{B}}+\varepsilon^{\alpha\,\hat{a}\check{b}}\hat{\nabla}^0_\alpha q^+_{\check{b}\,A\underline{B}}\,,\quad \delta_\varepsilon q^{+\check{a}}_{A\underline{B}}=-\varepsilon^\alpha\hat{\nabla}^0_\alpha q^{+\check{a}}_{A\underline{B}}-\varepsilon^{\alpha\,\hat{b}\check{a}}\hat{\nabla}^0_\alpha q^+_{\hat{b}\,A\underline{B}}\,,\\
\delta_\varepsilon(V^{++}_L)_{AB} &= -\frac{4i\pi}{k}(\varepsilon^\alpha\theta^0_\alpha)\left(q^{+\check{a}}_{B\underline{A}}q^{+\underline{A}}_{\check{a}A}-q^{+\hat{a}}_{B\underline{A}}q^{+\underline{A}}_{\hat{a}A}\right)+\frac{8i\pi}{k}(\varepsilon^{\alpha\hat{a}\check{b}}\theta^0_\alpha)\,q^{+\underline{B}}_{\hat{a}(A}\,q^+_{\check{b}\,B)\underline{B}}\,,\\
\delta_\varepsilon(V^{++}_R)_{\underline{AB}} &= \frac{4i\pi}{k}(\varepsilon^\alpha\theta^0_\alpha)\left(q^{+\hat{a}}_{A\underline{B}}q^{+A}_{\hat{a}\,\underline{A}}-q^{+\check{a}}_{A\underline{B}}q^{+A}_{\check{a}\,\underline{A}}\right)-\frac{8i\pi}{k}(\varepsilon^{\alpha\hat{a}\check{b}}\theta^0_\alpha)\,q^{+B}_{\hat{a}(\underline{A}}\,q^+_{\check{b}B\underline{B})}\,.
\end{aligned} \tag{5.12}$$

Together with the manifest $\mathcal{N}$=3 supersymmetry these five extra ones form the $\mathcal{N}$=8 supersymmetry, with an on-shell closure. Since the action (5.4) is $\mathcal{N}$=3 superconformal, it is also $\mathcal{N}$=8 superconformal.

We close this section with two comments.

First, the reason why the models with the $\mathrm{USp}(2N)\times\mathrm{USp}(2M)$ gauge group have neither additional supersymmetries nor full SO(8) R-symmetry, despite their formal resemblance to the $\mathrm{SU}(2)\times\mathrm{SU}(2)$ model, is that the conditions (5.9) are not satisfied in the generic $N > 1, M > 1$ case. The choice of $N = M = 1$ is the unique option when they are valid. Thus the only additional internal symmetry of the $\mathrm{USp}(2N) \times \mathrm{USp}(2M)$ models (extending the manifest SO(4) one) is the SO(5)/SO(4) symmetry (5.6) commuting with the $\mathcal{N}$=3 supersymmetry and not affecting the gauge superfields at all.

Secondly, it is a consistent reduction to put

$$\text{(a)}\ \ q^{+\hat{a}}_{11} = q^{+\hat{a}}_{22} = q^{+\check{a}}_{11} = q^{+\check{a}}_{22} = 0 \quad \text{or} \quad \text{(b)}\ \ q^{+\hat{a}}_{12} = q^{+\hat{a}}_{21} = q^{+\check{a}}_{12} = q^{+\check{a}}_{21} = 0\,. \tag{5.13}$$

These conditions break the gauge group $\mathrm{SU}(2) \times \mathrm{SU}(2)$ down to its subgroup $\mathrm{U}(1) \times \mathrm{U}(1)$. Actually, two options in (5.13) are equivalent to each other and one can focus on (5.13a). In this case one is left with four independent hypermultiplets $q^{+\hat{a}}_{12}, q^{+\hat{a}}_{21}$ and $q^{+\check{a}}_{12}, q^{+\check{a}}_{21}$ as compared with eight such hypermultiplets in the $\mathrm{SU}(2) \times \mathrm{SU}(2)$ case and two hypermultiplets in the minimal $\mathrm{U}(1) \times \mathrm{U}(1)$ case considered in Subsection 3.2. The numbers of real scalar fields in these models are, respectively, 16, 32 and 8. The doubling of hypermultiplets as compared to the minimal $\mathrm{U}(1) \times \mathrm{U}(1)$ case allows one to preserve the $\mathrm{SU}(2)_{\mathrm{PG}}$ multiplet structure and to retain all properties of the "parent" $\mathrm{SU}(2) \times \mathrm{SU}(2)$ model: the $\mathcal{N}$=8 supersymmetry and SO(8) R-symmetry. The corresponding transformations can be obtained from the above $\mathrm{SU}(2) \times \mathrm{SU}(2)$ ones by performing there the reduction (5.13a). Note that the opportunity to obtain the $\mathcal{N}$=8 supersymmetric $\mathrm{U}(1) \times \mathrm{U}(1)$ model through such a reduction of the ABJM $\mathrm{SU}(2) \times \mathrm{SU}(2)$ model was previously noticed in [30].

Finally, we would like to point out that it is still an open question whether any other gauge $\mathcal{N}$=3 Chern-Simons-matter model with $\mathcal{N}$=8 supersymmetry can be explicitly constructed. The $\mathcal{N}$=3 harmonic formalism seems to be most appropriate for performing such an analysis, since within its framework the issue of existence of one or another hidden symmetry amounts to examining simple conditions under which unwanted quartic contributions to the full variation of the hypermultiplet parts of the total action are vanishing.

6 Discussion

In this paper we gave a new superfield formulation of the ABJM theory with gauge groups $\mathrm{U}(N) \times \mathrm{U}(N)$ and $\mathrm{SU}(N) \times \mathrm{SU}(N)$ as well as of some its generalizations, in the harmonic $\mathcal{N}$=3, d=3 superspace where three d=3 supersymmetries are manifest and off-shell. We found the $\mathcal{N}$=3 superfield realization of the hidden $\mathcal{N}$=6 supersymmetry and SO(6) R-symmetry of the ABJM theory and demonstrated how these symmetries are enhanced to $\mathcal{N}$=8 and SO(8) in the BLG case of the gauge group $\mathrm{SU}(2) \times \mathrm{SU}(2)$. We also presented an example where $\mathcal{N}$=6 supersymmetry and SO(6) R-symmetry are reduced to $\mathcal{N}$=5 and SO(5), respectively. One of the salient features of the $\mathcal{N}$=3 formulation is that its superfield equations of motion are written solely in terms of analytic $\mathcal{N}$=3 superfields and have a surprisingly simple form, see (3.21), (3.46) and (3.47). Another nice property is that the invariant actions are always represented by the difference of the $\mathcal{N}$=3 superfield Chern-Simons actions for the left and right gauge groups plus the actions of two hypermultiplets which sit in the bifundamental representation of the gauge group and are minimally coupled to the gauge superfields. No explicit superfield potential is present in the action, as is dictated by the $\mathcal{N}$=3 superconformal invariance. The famous sextic scalar potential of the

JHEP03(2009)096

component formulation naturally emerges on shell as a result of the elimination of some auxiliary degrees of freedom from the gauge and hypermultiplet superfields. The $\mathcal{N}$=3 superfield formulation suggests a simple technical criterion as to whether a chosen gauge group admits the existence of hidden additional supersymmetries and R-symmetries: it is the cancellation of the terms quartic in the hypermultiplets in the full variation of the gauge-covariantized hypermultiplet action.

To clarify the significance of the $\mathcal{N}$=3 superfield formulation presented here, let us resort to the analogy between the ABJM theory and the $\mathcal{N}$=4, d=4 super Yang-Mills (SYM_4^4) theory, which describe the low-energy dynamics of multiple M2 and D3 branes, respectively. As is well known, the SYM_4^4 model is the maximally supersymmetric and superconformal gauge theory in four dimensions, a fact crucial for the string theory / field theory correspondence (see e.g. [48]). The $\mathcal{N}$=2, d=4 harmonic superspace [42] provides the appropriate off-shell $\mathcal{N}$=2 superfield description of SYM_4^4 as SYM_4^2 plus an $\mathcal{N}$=2 hypermultiplet in the adjoint representation minimally coupled to the $\mathcal{N}$=2 gauge superfield. Such a formulation was successfully used to study the low-energy quantum effective action and the correlation functions of composite operators in $\mathcal{N}$=2 superspace (see, e.g., [49] and [50]).

Analogously to SYM_4^4, the ABJM model is the maximally supersymmetric and superconformal Chern-Simons-matter theory in three dimensions.[7] The ABJM construction opened up ways for studying the $\mathrm{AdS}_4/\mathrm{CFT}_3$ correspondence between three-dimensional field models and four-dimensional supergravity in AdS space [5]–[21]. We believe that the $\mathcal{N}$=3 superfield description of the ABJM model and its generalizations developed in the present paper will be as useful for studying their algebraic and quantum structure as the $\mathcal{N}$=2 harmonic superspace approach has proved to be for SYM_4^4. In particular, we expect that it will be very efficient for investigating the low-energy quantum effective action in three-dimensional $\mathcal{N}$=6 supersymmetric field models as well as for computing the correlation functions of composite operators directly in $\mathcal{N}=3, d=3$ harmonic superspace, because the manifest off-shell $\mathcal{N}$=3 supersymmetry is respected at each step of the computation.

Furthermore, there are also other directions for extending the present study. A natural generalization is to find the $\mathcal{N}$=3 superfield description for superconformal field models with $\mathcal{N}$=4 and $\mathcal{N}$=5 supersymmetries, which are also interesting from the point of view of the AdS/CFT correspondence. We already considered one such example in Subsection 4.3. Another evident task is the coupling of the ABJM $\mathcal{N}$=3 superfield models to (conformal) $\mathcal{N}$=3 superfield supergravity.

As one more possible development, one may hope that our $\mathcal{N}$=3 superfield reformulation is capable to give further insight into the structure of those BLG theories which are based on the Nambu bracket (see [34]–[36]) and to clarify their relation to the M5 brane. In this connection, we mention that the equations of motion in the analytic $\mathcal{N}$=3 superspace (3.46) and (3.47) for the $\mathrm{U}(N) \times \mathrm{U}(M)$ model (and their analogs for the other models considered) possess an equivalent formulation in ordinary $\mathcal{N}$=3 superspace

[7] Well, almost: The maximal supersymmetry in three dimensions with a highest spin of one is $\mathcal{N}$=8, corresponding to the BLG special case of the ABJM model. However, the BLG model describes only *two* M2 branes since it is based on $\mathrm{SU}(2) \times \mathrm{SU}(2)$ while the ABJM model serves perfectly for an *arbitrary* number of M2 branes since it is based on $\mathrm{U}(N) \times \mathrm{U}(N)$.

JHEP03(2009)096

as follows. Using the bridges for the gauge superfields and passing to the central basis in $\mathcal{N}$=3 harmonic superspace and the so-called τ gauge frame [43], one can convert the equations (3.46) to the form of flat harmonicity conditions, which imply that the corresponding hypermultiplet superfields are linear in the harmonics u_i^+. At the same time, the spinorial harmonic analyticity conditions become highly nonlinear in this case, and one may think that the τ-frame form of the Chern-Simons equation (3.47) arises as an integrability condition for these nonlinear harmonic analyticity constraints. At this point there might be contact with a recent paper [35], where the equations of motion for the Nambu-bracket BLG theory were rewritten in terms of $\mathcal{N}$=8 superfields as some superfield constraint of first order in a gauge-covariantized spinor derivative. Based on the analogy with the ordinary $\mathcal{N}$=3 superfield form of the ABJM equations just mentioned, we guess that the constraint of [35] can be interpreted as a kind of Grassmann harmonic $\mathcal{N}$=8 analyticity in the τ frame.

Finally, it is worthwhile to note that the interrelations between the low-energy actions describing M2 and D2 branes was the subject of many papers (see, e.g., [37]). It was discovered that this issue is intimately related to a new type of Higgs phenomenon. It is clearly of interest to elaborate on it using our $\mathcal{N}$=3 superfield framework. In the appendix we show how this phenomenon arises in the simplest U(1) × U(1) model of Subsection 3.2.

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A Higgs effect in the U(1) × U(1) model

Here we briefly discuss how the Higgs-type effect of refs. [37] arises in the framework of the $\mathcal{N}$=3 superfield formalism. We shall consider the simplest U(1) × U(1) model of Subsection 3.2. The corresponding superfield action, the sum of (3.17) and (3.18), can be treated as a low-energy limit of the worldvolume action of single M2 brane.

The gauge group (3.19) which acts on hypermultiplets is realized by the following infinitesimal transformations

$$\delta q^{+a} = \Lambda\, q^{+a}\,, \quad \delta \bar{q}^{+}_{a} = -\Lambda\, \bar{q}^{+}_{a}\,, \quad \delta A^{++} = -D^{++}\Lambda\,, \quad \Lambda = \Lambda_L - \Lambda_R\,. \tag{A.1}$$

The rest of the gauge $\mathrm{U}(1)\times\mathrm{U}(1)$ group with the parameter $\hat\Lambda = \Lambda_L + \Lambda_R$, acts only on V^{++}: $\delta V^{++} = -D^{++}\hat\Lambda\,.$

As the first step we pass in (3.18) to the dual ω, f^{++} description by decomposing

$$q^{+a} = u^{+a}\omega - u^{-a} f^{++}\,, \quad \bar{q}^{+}_{a} = -u^{+}_{a}\tilde\omega + u^{-}_{a}\tilde f^{++}\,. \tag{A.2}$$

Assuming that there is a constant real condensate in ω,

$$\omega = c_0 + \hat\omega\,, \quad \bar{c_0} = c_0\,, \tag{A.3}$$

and taking into account the gauge transformation law

$$\delta\hat\omega = \Lambda\,(c_0 + \hat\omega)\,, \tag{A.4}$$

one can choose the "unitary" gauge in which the imaginary part of $\hat\omega$ has been completely gauged away:

$$\widetilde{\omega} = \omega\,, \quad \widetilde{\hat\omega} = \hat\omega\,. \tag{A.5}$$

Up to a total harmonic derivative, the Lagrangian in the action (3.18) in this gauge is rewritten as

$$\mathcal{L}_q = \left(f^{++} + \tilde f^{++}\right) D^{++}\hat\omega - f^{++}\tilde f^{++} - A^{++}\left(f^{++} - \tilde f^{++}\right)(c_0 + \hat\omega). \tag{A.6}$$

Upon varying with respect to the auxiliary superfields $f^{++}, \tilde f^{++}$ and substituting the result back into (A.6), we obtain

$$\mathcal{L}_q \;\Rightarrow \tilde{\mathcal{L}}_q \;= (D^{++}\hat\omega)^2 - (c_0 + \hat\omega)^2 (A^{++})^2\,. \tag{A.7}$$

We see that the superfield A^{++} is now also auxiliary and can be eliminated from the sum $S_{\rm gauge} + S_{\rm hyp}$, eqs. (3.17), (3.18), by using its algebraic equation of motion

$$A^{++} = -\frac{ik}{16\pi}\,\frac{1}{(c_0 + \hat\omega)^2}\,W^{++}(V). \tag{A.8}$$

Substituting this expression back into the total action, we obtain

$$S_{\rm gauge} + S_{\rm hyp} \;\Rightarrow\; \int d\zeta^{(-4)}\left[(D^{++}\hat\omega)^2 - \frac{k^2}{(16\pi)^2}\frac{1}{(c_0+\hat\omega)^2} W^{++}(V)W^{++}(V)\right]. \tag{A.9}$$

This action is a sum of the free real $\hat\omega$ hypermultiplet action and the $\mathcal{N}{=}3, d{=}3$ Maxwell action multiplied by the "dilaton" factor which ensures the (spontaneously broken) superconformal invariance of the final gauge-fixed action (recall that we started from the action invariant under the $\mathcal{N}{=}3, d{=}3$ superconformal symmetry). It should also be implicitly invariant under nonlinearly realized SO(6) symmetry and hidden $\mathcal{N}{=}3$ supersymmetry, since these invariances are inherent in the sum of the actions (3.17), (3.18) we started with. It is interesting to inquire what kind of nonlinear sigma model for scalar fields is hidden in (A.9). One has now four real scalar fields in the $\hat{\omega}$ hypermultiplet and three physical scalars in the gauge action (former auxiliary fields $\phi_L^{(kl)} + \phi_R^{(kl)}$ of the Chern-Simons superfield action), i.e. total of seven physical scalar fields.[8] One of these bosonic fields is dilaton, so there remain six bosonic fields which should support a nonlinear realization of the group SO(6) $\sim$ SU(4). The only 6-dimensional coset manifold of SU(4) seems to be $\mathbb{CP}^3 \sim$ SU(4)/U(3), so we expect that the action (A.9) contains the d=3 nonlinear $\mathbb{CP}^3$ sigma model in its bosonic sector and thus can be interpreted as a low-energy limit of the single D2 brane action on $AdS_4 \times \mathbb{CP}^3$.

It would be interesting to see how the above procedure generalizes to the non-Abelian case.

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